Principles of Influence Line Analysis

An Online Continuing Education Course for Engineers

Course Number: S-4013

Credit: 4 Hours / 4 PDH / 4 CPD

Principles of Influence Line Analysis

Brian A. Neiss, P.E.

INTRODUCTION

When structures are analyzed, one of the primary objectives is to determine the most severe responses to loading. Certain regions of a structure, such as fatigue-sensitive connections, especially require attention. In the design process, conventional wisdom typically leads the engineer to simply check a few load patterns and conclude the analysis. In most cases this scope of work may be sufficient, but may raise some lingering questions:

- How can one prove that a maximum or minimum structural response is considered?
- How many locations in the structure are critical?
- How many load patterns should be considered?
- Are the loads live and/or moveable?

The answers to these questions are not always clear, and these issues become especially challenging when analyzing complex structures. The reality is that every structure has an unbounded set of load patterns to consider at each location. It is obvious from a business perspective that time and resources are not available to generate force diagrams for every possible load pattern at every location. Therefore, there is a need for an organized and efficient methodology to address the fundamental question of how to position loads to create the most critical response. Influence line analysis addresses this concern.

This course first introduces the concept of the influence line and briefly reviews the theoretical background behind it. Influence lines of statically determinate structures and statically indeterminate structures are then covered in detail using numerous examples, including a practical design application. For computer-based solutions of influence lines, verification of influence line output is also demonstrated in an example.

WHAT IS AN INFLUENCE LINE?

An influence line is a graphical mapping or variation of a structural response (i.e., moment, shear, reaction, axial force) as an imaginary "unit" load travels at any point in the structure. It may be characterized as a virtual deformed shape and is specific to a certain location in the structure.

An internal force diagram can capture the shear, bending moment, or axial load at every location in the structure, but it is specific to a fixed load pattern. With an influence line for a response to the same structure, the engineer can easily determine many responses caused by this same load pattern moved to any location. Unlike internal force diagrams, an influence line is only applicable to a specific location in the structure and therefore several influence lines may be necessary for analysis to capture the most critical regions of the structure.

The table below explains the key differences between a response (moment, shear, deflection, etc.) diagram and influence line:

Table: Comparison between Response Diagram and Influence Line

Response Diagram (Moment, Shear, Axial Load, etc.)	**Influence Line**
Load pattern is fixed in place	Load pattern can move anywhere in the structure
Analyze *actual* structural response directly	Analyze a *virtual* deformed shape as an intermediate step
Only one pattern is analyzed at a time	Many load patterns may be efficiently analyzed at a time
Structural response encompasses entire structure	Structural response is specific to a certain location

MULLER-BRESLAU PRINCIPLE

The Müller-Breslau principle is useful for generating qualitative influence lines. It demonstrates that upon releasing a constraint and imposing a virtual displacement at the same location in a structure, the resulting deflected shape of the structure forms its influence line. This constraint release could take the form of a support movement or a section cut in the structure. Consider for instance a pin and roller supported beam supported at A and B with a free end at C. Span lengths L_1 and L_2 are defined as shown:

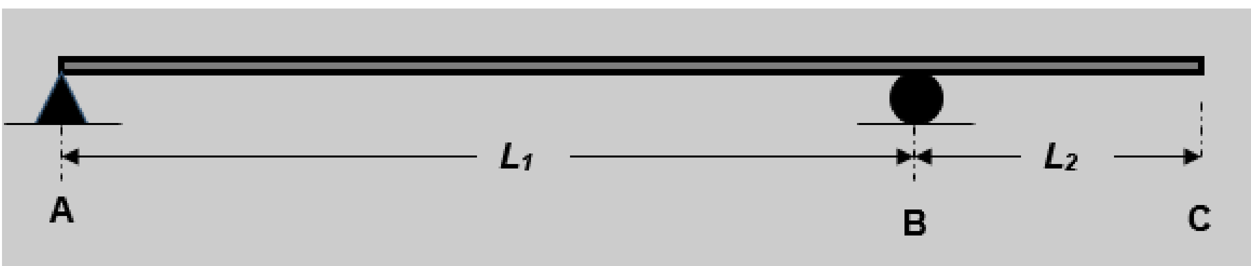


Figure: Simply Supported Beam Arrangement

If a generalized influence line for the reaction at A is sought, the support at A is removed and a displacement of Δ is imposed. The whole beam undergoes a virtual deflection shown:

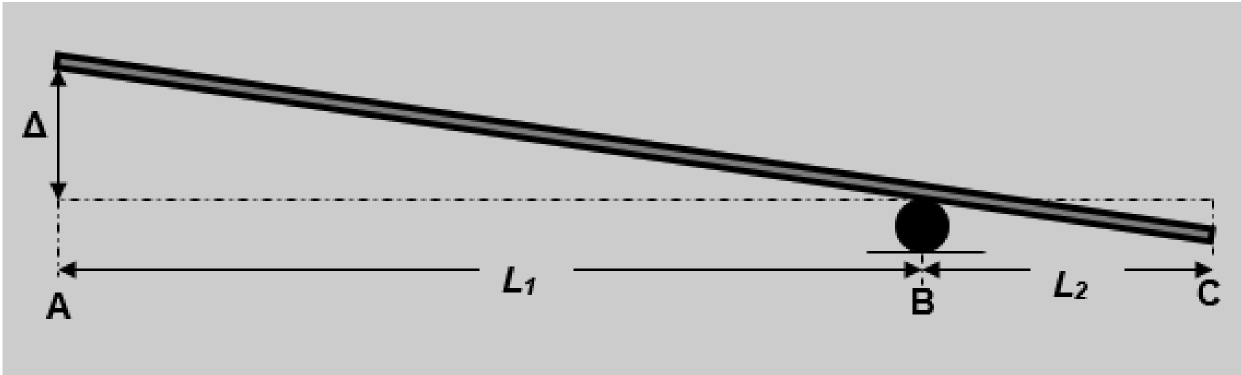


Figure: Virtual Deflection of Simply Supported Beam

The resulting deflected shape maintains the remaining translational fixity at "B," (zero displacement). At this stage, the virtual deflection at any point of the beam can be calculated based on the deflection at A.

This principle draws upon the principle of virtual work. If a structure loaded with forces is in equilibrium and subsequently undergoes a virtual displacement, the virtual work of these forces is zero.

BETTI'S THEOREM

Betti's Theorem provides a technical background for influence line analysis from a quantitative standpoint. Consider two different static load patterns acting on a structure, P and Q each causing deformed shapes or displacements of p and q, respectively. Betti's Theorem shows that the work produced by the product of forces P and the displacements q is equal to the work produced by the product of forces Q and displacements p. Mathematically, Betti's Theorem may be represented as follows:

$$\sum_{i=1}^{n} P_i\, q_i = \sum_{j=1}^{m} Q_j\, p_j$$

As it relates to influence line analysis, a less generalized form of Betti's theorem is developed by considering actual external live or moving loads on a structure, the corresponding deflections generated by using the Müller-Breslau principle and the virtual work required to cause the "break" in the continuity in the structure.

Example:

In the statically determinate beam shown below, use the Müller-Breslau principle and Betti's Theorem to determine the internal shear force at point A, based on a unit load placed at point B.

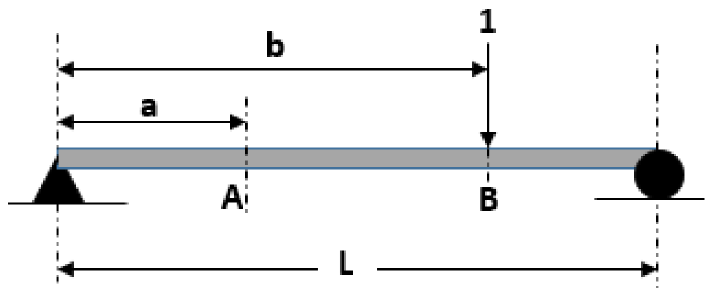


Figure: Simply Supported Beam with Traversing Unit Load

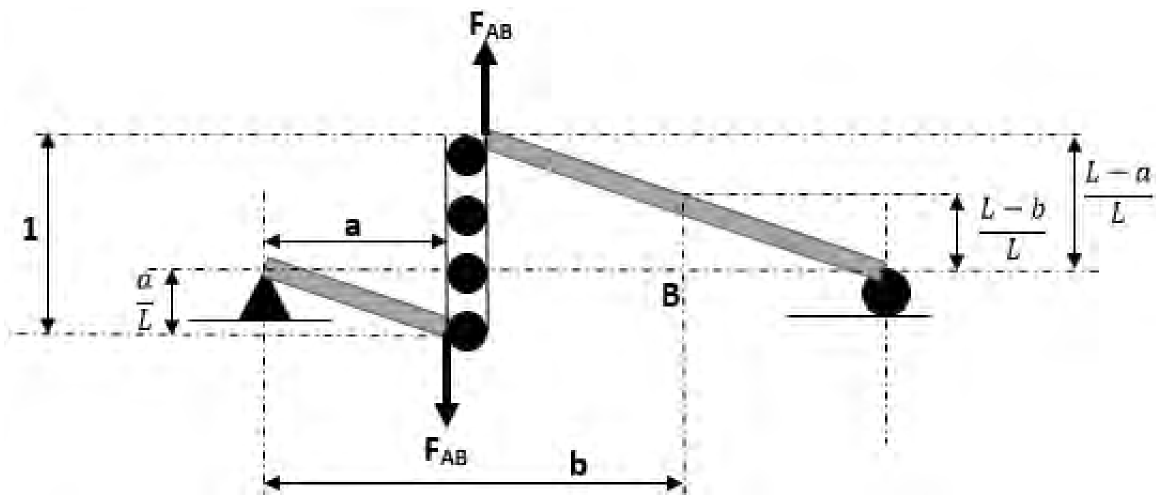


Figure: Simply Supported Beam with Virtual Unit Displacement by Virtual Shear Load F_{AB}

Taking the example in the figure above, the Müller-Breslau principle is applied to the simply-supported beam. The section cut shears the beam exactly one unit at point A. At the section cut, the idealization is such that the virtual displacements on either side are guided by rollers.

Betti's Theorem is applied to solve for the shear force V_{AB}, where:

- The set of P contains actual loads (i.e., -1 and V_{AB}).
- The set of Q contains virtual shear force F_{AB}, acting at point A.
- The set of p contains zero relative displacement at point A.
- The set of q maps out a virtual deflected shape caused by F_{AB}.

$$\sum_{i=1}^{n} P_i\, q_i = \sum_{j=1}^{m} Q_j\, p_j$$

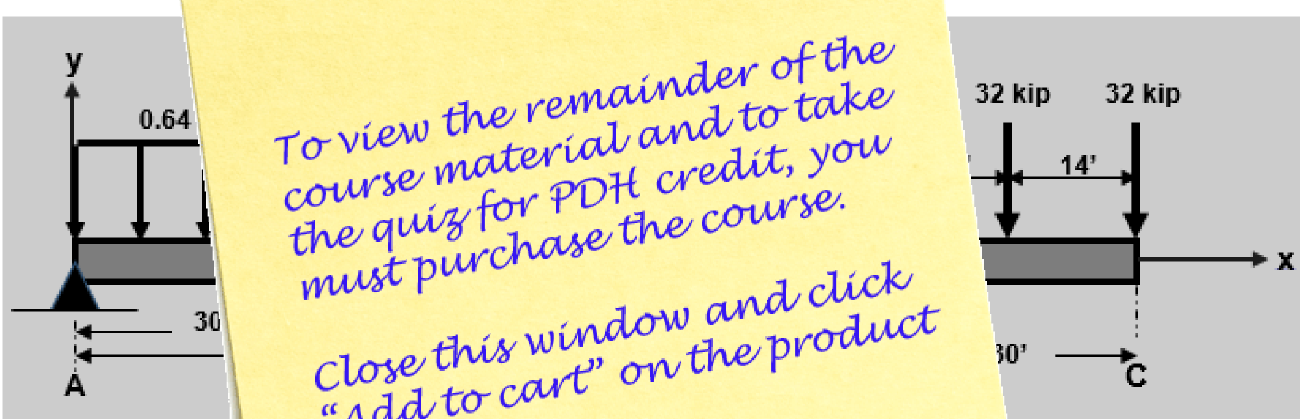
$$V_{AB}\,(1) + (-1)\frac{L-b}{L} = F_{AB}\,(0)$$

$$V_{AB}\,(1) = (1)\frac{L-b}{L}$$

$$V_{AB} = \frac{L-b}{L}, (solution)$$

Numerical Example:

A simply supported beam with a fi... Betti's Theorem and the Müller-Bresla...



A virtual displacen... d. The influence function specified f... lacement (support movement... at x=90 ft remains at zero and...