# Optimum Design of Indeterminate Beams

An Online Continuing Education Course for Engineers

**Course Number: S-3003**

**Credit: 3 Hours / 3 PDH / 3 CPD**

© 2023 Decatur Professional Development, LLC. All rights reserved.

# Optimum Design of Indeterminate Beams

*Many of the concepts and equations developed, herein, are from W.B Readey's published work in the Journal of Aeronautical Sciences*

**Course Overview**

The design and analysis of indeterminate frames and beams are important problems encountered by all structural engineers. With modern computer programs, analysis is not a difficult problem. Design is more complex and less amenable to programming. For efficient design, something between the following two extremes is needed:

1) The first intuitive approximation of the structural design engineer, without any additional information, and
2) A complex computer program where, of necessity, the original programmer has made general decisions/assumptions which seriously impact the diverse design criteria for which it is used, thereby constraining those who use the program.

This course is designed to assist structural engineers from all fields - civil to aeronautical - in developing initial design concepts. This is accomplished by determining an idealized optimum section property distribution for each of the most critical loading conditions. Even though, for many large structures encountered by civil engineers, it is impractical to vary the section properties continuously in accordance with the idealized optimum, it is very helpful to know what that idealized distribution is, so that it can be approximated within the limits of practicality.

Because the design of Optimum Beams - considered in this course - is a special case of the general theory, "Optimum Design of Indeterminate Frames", the general theory is briefly covered in this course.

The student is assumed to be familiar with basic strength analysis theory and elementary calculus.

**Introduction**

Sometimes an indeterminate structure is designed by arbitrary methods and then checked for strength, with no assurance that the resulting structure is highly efficient.

This course presents a method, with examples, for obtaining idealized, optimum section property distributions - directly from design conditions - for the special case of constant depth, indeterminate beams.


©2011 Decatur Professional Development, LLC All rights reserved

For many, the wonders of modern, computer analysis programs, and the over dependence upon them, have been accompanied by a gradual, almost insidious, loss of understanding of the fundamental concepts upon which those analyses depend.

This course is designed to address that problem.

The first section includes a mini analysis, which covers the basic optimum design concepts. It uses unsophisticated, intuitive logic and descriptive diagrams.

This is followed by a brief section: "Determination of Section Properties for Idealized Optimum Beams".

Then, after the intuitive understanding has been fortified, the description and limitations of the general method are covered more conventionally and the formal analysis is presented.

**Basic Optimum Design Concepts & Unsophisticated, Intuitive, Mini Analysis**

The basic concepts of idealized optimum design are the essence of simplicity. From one end of the beam to the other, the extreme fiber stresses - tension and compression - are designed to be equal to the maximum allowable tension and compression stresses, respectively.

To illustrate, consider a constant depth beam, of length L, fixed at both ends and subjected to stresses in the elastic range.

Assume that, as usual, the loading is such that there are two inflection points.

Because each of the extreme-fiber stresses, tension and compression, remain constant over the entire length of the beam, the curvature of the beam - for both convex and concave sections - is constant.

Because the fixed ends are parallel, the total change in slope is zero; therefore, the sum of the lengths of the convex sections must equal the sum of the lengths of the concave sections.

Thus, the length of the center section (concave for down loads and convex for up loads) must be equal to the sum of the lengths of the end sections which, because of geometric considerations, must be equal in length.

Therefore, the inflection points are at .25 L and .75L.

Note that, the locations of the inflection points have been determined before either the specific loading or the section properties have been specified; and what was an indeterminate beam, has now become a determinate idealized beam.


©2011 Decatur Professional Development, LLC All rights reserved

The idealized beam is equivalent to a center beam supported at its ends by two cantilever beams - each half the length of the center beam.

Now, the optimum section property distribution for each of the two or three most critical loading conditions could be determined quickly.

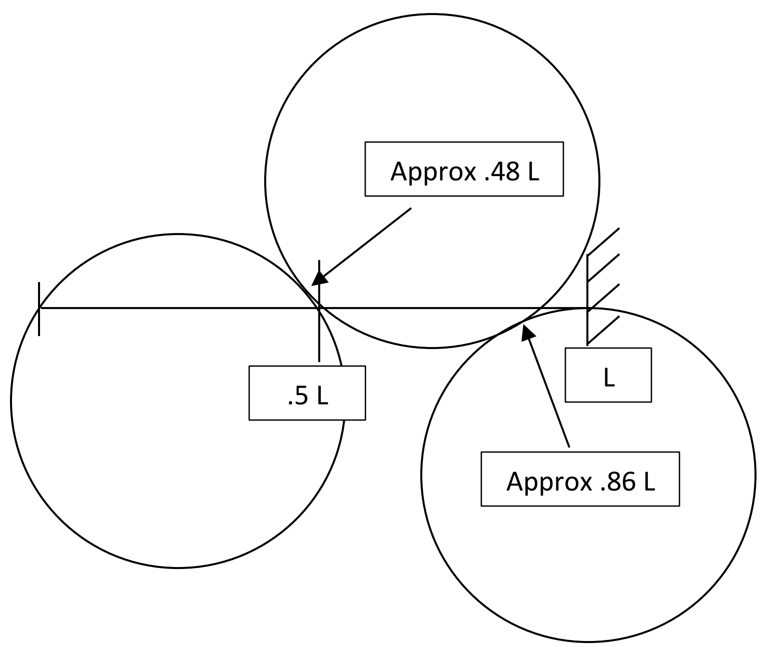
With this information, and considerations of other practical problems and constraints, a good first-cut design could be created by the designer, and then analyzed by conventional methods.

Having the same infection points - for the idealized beams designed for each of the critical loading conditions - simplifies the determination of an efficient "real-life" section property distribution that can withstand all of the required conditions.

Now to get a better intuitive insight into optimum design, let's look at a second problem which, also, will be covered in more detail, later in this course, in the Formal Analysis.

Consider a constant depth beam with two equal length spans. The beam is fixed at the right end, with simple supports at the left end and center.

Again, because this idealized beam bends in arcs of constant curvature, simple geometric considerations can be used to determine the locations of the inflection points, as shown in the following figure:



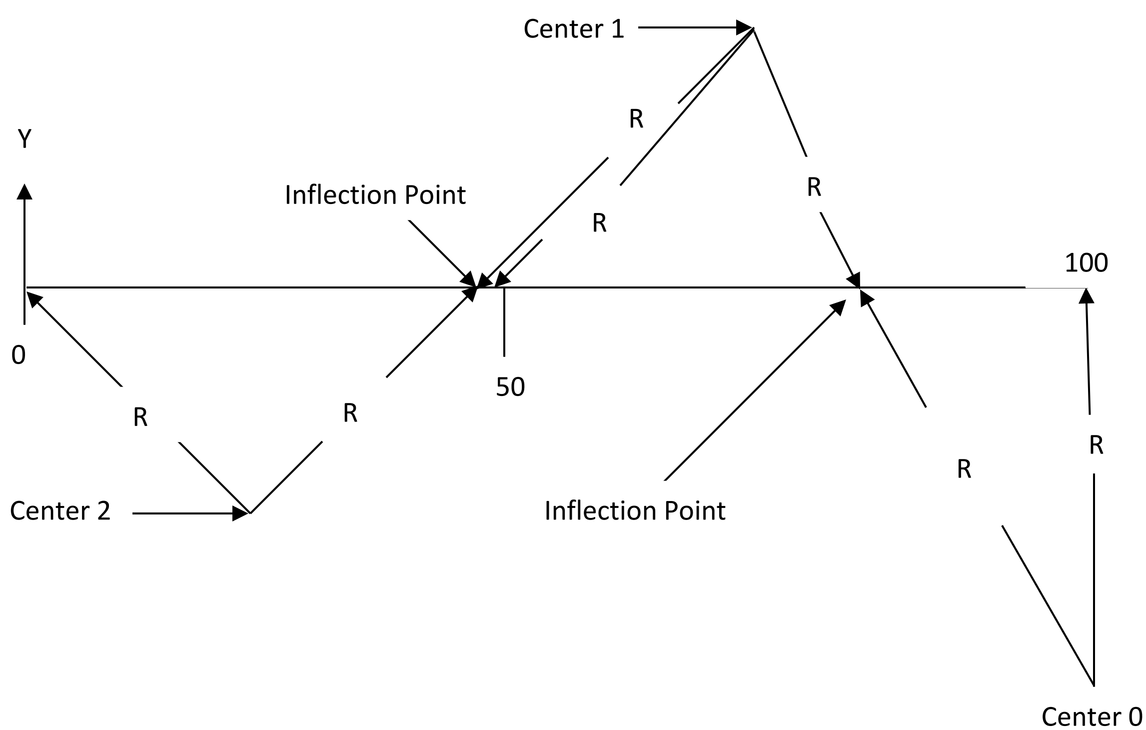


©2011 Decatur Professional Development, LLC All rights reserved

Just to gain a better intuitive understanding of this concept, it is suggested that the student estimate the inflection point locations by drawing a construction similar to the above figure.

Using a much larger radius would determine the sensitivity of the estimates to the size of the radius.

More precise inflection point locations can be calculated using the sketch below:




**Optimum Beam with Constant Radius of Curvature**

Center 1 is located on the circles with equations:

$$y + R = ((2R)^2 - (x - 100)^2)^{0.5} \qquad \&$$
$$y = (R^2 - (x - 50)^2)^{0.5}$$

If a value for, "R" is arbitrarily chosen, there are two equations in two unknowns.

In those events where this process yields equations that are difficult to solve, and only a laptop computer is available, it is convenient to have an alternate spreadsheet method available as shown below:


©2011 Decatur Professional Development, LLC All rights reserved

Calculating y values for Circle 1 (Radius = 2R with center at x = 100 and y = - R) and Circle 2 (Radius = R with center at x = 50 and y = 0)

$$y + R = ((2R)^2 - (x - 100)^2)^{0.5}$$

$$y = (R^2 - (x - 50)^2)^{0.5}$$

| Circle 1 | | Circle 2 | | | |
|---|---|---|---|---|---|
| x | y | Xip | y | R0 = | 200 |
| 70.5 | 198.9107 | | 198.9466 | | |
| 70.52 | 198.9122 | | 198.9445 | | |
| 70.54 | 198.9137 | | 198.9425 | | |
| 70.56 | 198.9151 | | 198.9404 | | |
| 70.58 | 198.9166 | | 198.9383 | | |
| 70.6 | 198.9181 | | 198.9363 | | |
| 70.62 | 198.9196 | | 198.9342 | | |
| 70.64 | 198.921 | | 198.9321 | | |
| 70.66 | 198.9225 | | 198.93 | | |
| 70.68 | 198.924 | | 198.928 | | |
| 70.7 | 198.9254 | 85.35 | 198.9259 | | |
| 70.72 | 198.9269 | | 198.9238 | | |
| 70.74 | 198.9284 | | 198.9217 | | |
| 70.76 | 198.9298 | | 198.9196 | | |
| 70.78 | 198.9313 | | 198.9175 | | |
| 70.8 | 198.9328 | | 198.9155 | | |
| 70.82 | 198.9342 | | 198.9134 | | |


©2011 Decatur Professional Development, LLC All rights reserved

| | | | | | |
|---|---|---|---|---|---|
| 70.84 | 198.9357 | | 198.9113 | | |

0.5*(100+70.7)

shows that the inflection point is located at 85.35% of the beam length, L. The inflection point is at the midpoint between centers 0 and 1.

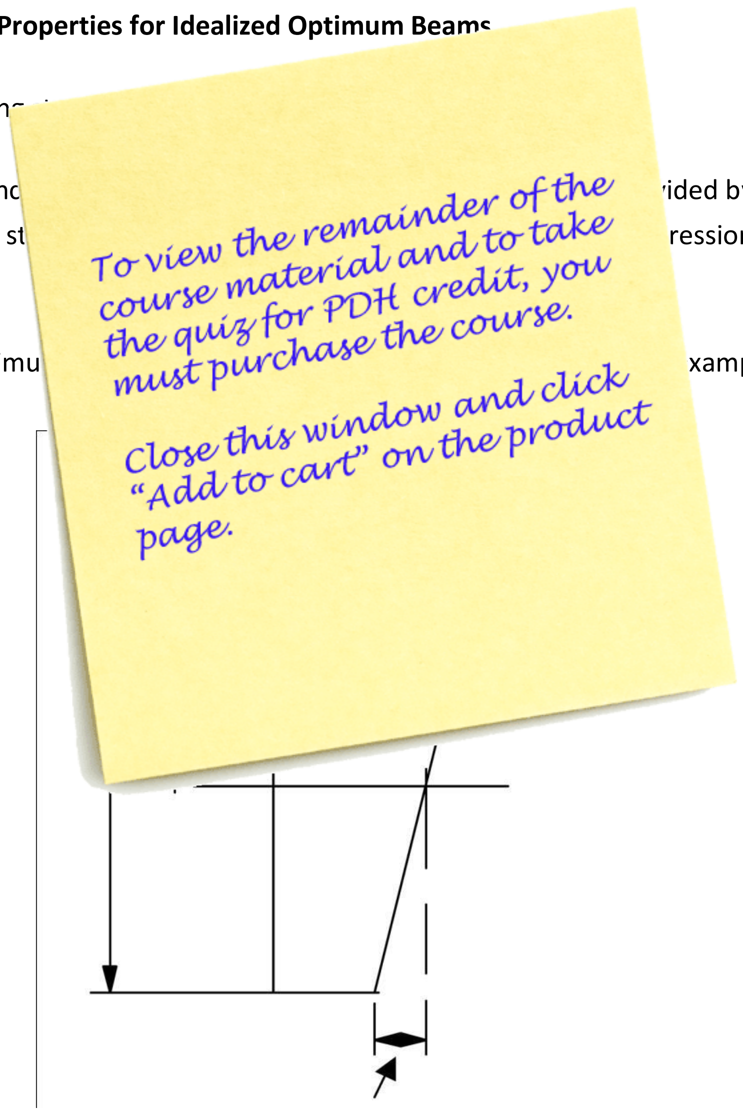
Obviously, the other inflection point can be calculated by the same procedure.

**Determination of Section Properties for Idealized Optimum Beams**

Fig. 1 shows a typical 1" lon

The angle of rotation per inc ... vided by H, where $\varepsilon_T$ = $\varepsilon_i$ - $\varepsilon_o$; and $\varepsilon_o$ and $\varepsilon_i$ are st ... ression strains are considered negative.

Constant, Maximu ... xample).




Constant, Maximum Allowable, Extreme Fiber Compression Strain ($\varepsilon_o$, in this example).

**Fig. 1 - Stressed Beam Section**


©2011 Decatur Professional Development, LLC All rights reserved