Structural Vibration Frequency Analysis of Beams

An Online Continuing Education Course for Engineers

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Compendium of Formulas for the Structural Vibration Frequency Analysis of Beams

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(Professional development hours = 3)

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1. INTRODUCTION

Fundamental natural frequency is an important parameter in structural dynamics analysis and applications. A number of simple formulas for the computation of the fundamental natural frequency of beam vibration under a variety of boundary conditions and loading are presented. Some are approximate formulas and some are exact. An estimate of the error is given for the approximate formulas whenever possible. The formulas presented here are simple enough that the fundamental frequency could be obtained by hand calculation without recourse to computers or finite element analysis. The formulas are demonstrated with nine worked out numerical examples. [Fundamental natural frequency is also referred as fundamental frequency. If a structure has more than one natural frequency the lowest frequency is the fundamental frequency.]

2. SINGLE-DEGREE-OF-FREEDOM SYSTEM

Beams have infinite degrees-of-freedom. However beams may be modeled as single-degree-freedom (SDOF) systems under certain conditions. This forms the basis for some of the formulas presented here. So we present in this section natural frequency computation of SDOF systems.

2.1 Frequency Analysis of Single-Degree-of-Freedom Systems

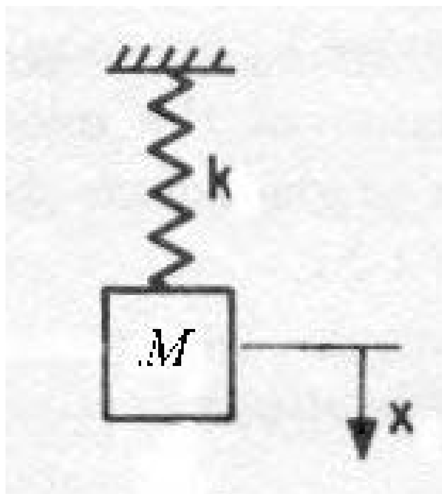

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Figure 1: Single-Degree-of-Freedom System Model for Frequency Analysis

k = stiffness,
M = mass, and
x = displacement of the mass

Natural frequency of the SDOF system in radians per second is given by

$$\omega = \sqrt{\frac{k}{M}} \tag{2.1}$$

Natural frequency in cycles per second is given by

$$f = \omega/2\pi \tag{2.2}$$

Period of the SDOF system in seconds is given by

$$\mathbf{T} = \frac{1}{f} \tag{2.3}$$

2.2 Static Deflection Approach

Equation (2.1) may be rewritten as

$$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{k}{(W/g)}} = \sqrt{\frac{kg}{W}} = \sqrt{\frac{g}{\Delta}} \qquad (2.4)$$

In the above equation we have substituted

M = W/g (mass = weight/acceleration due to gravity)

and

Δ = W/k (Static deflection due to weight = weight/stiffness)

Thus the formula for natural frequency in radians per seconds is,

$$\omega = \sqrt{\frac{g}{\Delta}} \qquad (2.5)$$

3. BEAM WITH A CONCENTRATED MASS

3.1 Simple Supported Beam

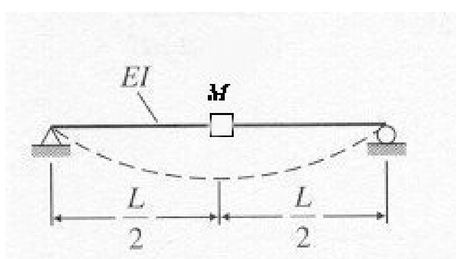

Figure 2: Simple Supported Beam with Concentrated Mass at Mid-span

Consider a uniform beam with both ends simple supported carrying a concentrated mass at mid-span (Figure 2). Distributed mass along the span (for example mass of the beam) is much smaller than the concentrated mass that we ignore the distributed mass in the frequency analysis. [What we mean by "much smaller" is discussed later in Section 3.]

Example of a concentrated mass is the mass of a machine placed at mid-span of the beam.

So, with only the single concentrated mass this beam can be modeled as an equivalent single-degree-of-freedom (SDOF) system as shown in Figure 1. Equivalent stiffness for this case is,

$$k = \frac{48EI}{L^3}$$

Mass of the SDOF is equal to the concentrated mass on the beam. How did we get the stiffness? We use the definition of stiffness. Stiffness is the force required to produce unit deflection. Alternately, stiffness is equal to force divided by deflection.

In order to compute the equivalent stiffness of the simple supported beam, apply a concentrated force (F) at mid-span and determine the deflection (Δ) at the mid-span.

$$\Delta = \frac{FL^3}{48EI}$$

Where E = Young's modulus of elasticity,
I = Moment of inertia of beam cross section, and
L = Span.

You may find this formula for static beam deflection in many structural engineering and engineering mechanics textbooks.

By definition, stiffness $k = \frac{Force}{Deflection} = \frac{F}{\Delta} = \frac{48EI}{L^3}$

Using eqn. (2.1), natural frequency ω of a uniform simple supported beam with a heavy concentrated mass at mid-span is given by

$$\omega^2 = \frac{k}{M} = \frac{48EI}{ML^3} \qquad (3.1)$$

3.2 Beams with Other Boundary Conditions

The frequency calculation approach used in Section 3.1 is applicable to beams with other boundary conditions also. As long as the beam is uniform and the distributed mass along the span is much smaller that than the concentrated mass at mid-span, eqn. (2.1) is applicable. Equivalent stiffness (k) for various boundary conditions are given in Table 1.

	Boundary Conditions	Equivalent Stiffness (k)
1.	Both ends simply supported	$\frac{48EI}{L^3}$
2.	Both ends clamped	$192EI$

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$\omega^2 = \frac{g}{\Delta}$

Static deflection due to concentrated weight, $\Delta = W/k$

where, k is the beam stiffness provided in Section 3.2. This will provide same natural frequency as Section 3.2. For example, if we use this formula for beams with both ends simple supported,