



# Earthquake Design of Equipment and Supports: Response Spectrum Method

An Online Continuing Education Course for Engineers

**Course Number: S-1006**

**Credit: 1 Hour / 1 PDH / 1 CPD**

# **Earthquake Design of Equipment and Supports: Response Spectrum Method**

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## 1. SCOPE

The purpose of this educational module is to present the response spectrum method of earthquake response analysis/design of mechanical and electrical equipment and supports. Equipment and supports are modeled as a single-degree-of-freedom (SDOF) system. No prior knowledge of structural vibration analysis or earthquake analysis is required to understand this module.

## 2. INTRODUCTION

Four methods of earthquake response analysis of equipment are available.

1. Simplified procedures such as those provided in ASCE 7-05 Standard [1]
2. SDOF response spectrum method
3. MDOF response spectrum method
4. Time-history method

SDOF - Single-degree-of-freedom

MDOF - Multiple-degrees-of-freedom

The SDOF response spectrum method, presented in this educational module, can be used as long as the equipment and supports can be modeled by a single-degree-of-freedom system consisting of a spring, mass and damping. The SDOF response spectrum method usually gives less conservative results than simplified procedures, yet is easy to perform. It is far easier to perform than MDOF response spectrum analysis or time-history analysis that require special computer software.

### 3. SDOF MODEL

The equipment and supports are modeled as a SDOF system as represented in Figure 1.

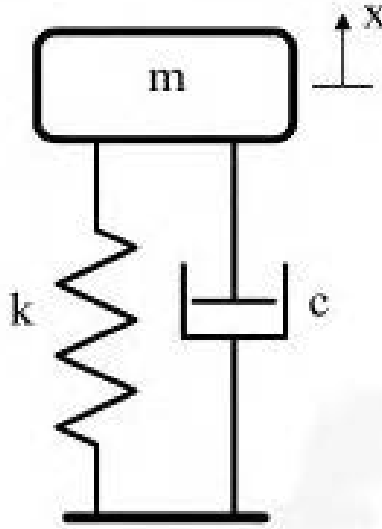


Figure 1: Single-Degree-of-Freedom System Model

$c$  = damping,  
 $k$  = stiffness,  
 $m$  = mass, and  
 $x$  = displacement of the mass

$k$  = stiffness of supports in the direction of the earthquake motion (If a horizontal earthquake motion is considered, then  $k$  is the corresponding horizontal stiffness of the supports; if vertical earthquake motion is considered, then  $k$  is the vertical stiffness of the supports.)

$$m = w/g \quad (1)$$

where  $w$  = weight of SDOF = weight of equipment + half the weight of supports  
 $g$  = acceleration due to gravity

Damping is often specified as a percentage of critical damping, or simply damping percentage, and denoted by  $\xi$ . As an example, 2% damping is often used for welded steel construction in some industries. Damping may also be presented as damping ratio, again denoted by  $\xi$ . If percentage of critical damping is 2%, then damping ratio is 0.02.

### 4. NATURAL FREQUENCY AND PERIOD OF THE SDOF SYSTEM

Natural frequency of the SDOF system in radians per second is given by

$$\omega^2 = k/m \quad (2)$$

Natural frequency in cycles per second is given by

$$f = \omega/2\pi \quad (3)$$

Cycles/second is also known as hertz.

Period of the SDOF system in seconds is given by

$$T = 1/f \quad (4)$$

### 5. RESPONSE SPECTRUM

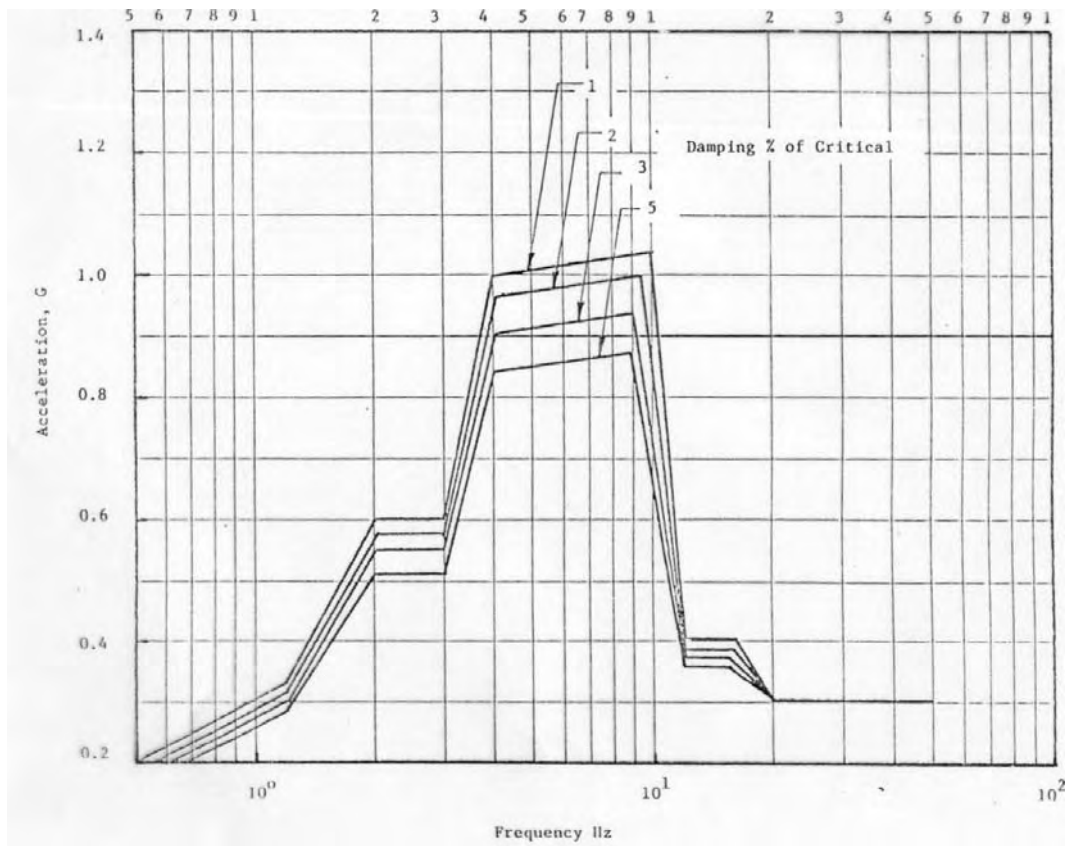


Figure 2: Sample acceleration response spectra

A family of acceleration response spectra for various damping values is shown in Figure 2. Horizontal axis is the frequency in hertz (hz) and vertical axis is the maximum acceleration of the mass as a multiple of g. This family of response spectra represents a given base motion of the SDOF system. (Note: Sometimes acceleration response spectra

are plotted as "Period versus maximum acceleration" instead of "frequency versus maximum acceleration". Velocity response spectra and displacement response spectra are also sometimes used. In those cases the vertical axis is maximum velocity or maximum displacement, as appropriate.)

If the equipment is located at the ground level, then the ground acceleration spectra are used in the analysis. There will be separate families of horizontal and vertical spectra corresponding to the horizontal and vertical earthquake motions.

If the equipment is located at a higher floor level, then the spectra for the earthquake motion at that level are used. These are called "input spectra" and are usually provided

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### 6.3 Maximum Force

Maximum force on the mass may be computed by multiplying mass by the maximum acceleration. Maximum acceleration should be specified in  $\text{inch}/\text{sec}^2$  or  $\text{feet}/\text{sec}^2$ .

$$\text{Maximum force} = \text{mass} \times \text{maximum acceleration} \quad (7)$$