Ranking Models for Decisions that Allow Attribute Trade-offs – SAW and TOPSIS

An Online Continuing Education Course for Engineers

Course Number: P-2023

Credit: 2 Hours / 2 PDH / 2 CPD

© 2023 Decatur Professional Development, LLC. All rights reserved.

Ranking Models for Decisions that Allow Attribute Trade-offs – SAW and TOPSIS

Deandra T. Cassone, PhD, PMP

1.0 Introduction

In corporate or government decision making, there typically are a number of attributes or characteristics associated with the alternatives that are presented for a decision. In many circumstances, these alternatives need to be screened or ranked. There are a number of different approaches and methods that can be used to capture the decision environment and provide structure to the decision-making process.

This course addresses decision making circumstances where the decision-maker is permitted to trade-off attribute values. In these models, changes in one attribute or the weighting associated with that attribute can be offset with changes in any other attribute. With models such as these, a single number is usually assigned to each multidimensional characterization representing an alternative.

This course will address two of these methods, the Simple Additive Weighting (SAW) method and the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) method. Both of these methods are widely used in industry and the government. Both the theory and application of these methods will be addressed in this course.

Topics discussed in this course include the following:

- Multiple Criteria Decision Making (MCDM) problems
- Compensatory decision models
- Construction of the decision matrix
- The theory behind SAW
- Application of the SAW method
- The theory behind the TOPSIS method
- Application of the TOPSIS method
- A comparison of the ranking of the methods presented

1.1 Multiple Criteria Decision Making

Decisions that rank alternatives based on several criteria measured with subjective and objective data are best modeled by multiple criteria decision-making methodologies. There is a large number of methods and variations of the methods that can be used to rank alternatives. Each method and its variation processes the data that is typically structured in a decision matrix in a different manner. Each of these methods has strengths and may be applied in different decision environments. Due to the nature of how these methods handle the data, they may position certain alternatives being higher ranking alternatives, based on how the processing occurs. As part of understanding the various methods, it is also important to understand the nature of the computations and how certain alternatives may be more favorable, based on the method chosen.

Portions ©2019 Decatur Professional Development, LLC All rights reserved.

Additionally, the data used in the decision is very important in the overall decision process. Subjective data is typically forward-looking when compared to objective data, which is historical or looking back. If some objective or historical data are readily available, this can be analyzed with statistics and used, along with the subjective data that is generated with expert opinion and used as input to the model. Numerous methods exist and can be applied to multiple criteria decision-making models, such as Simple Additive Weights (SAW) or the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), both of which are discussed later. These two methods are easiest to apply, the easiest to understand, and provide the best solution.

Examples of multiple criteria decision-making models may include the following:

- Selection of executives for promotion or retirement based on performance, evaluation criteria, and needs of the organization.
- Ranking critical items for the military based on their contributions to operation plans, readiness, sustainability, and availability.
- Selecting products to keep or delete in a product line based on sales volume, future potential sales, strategic importance, and their impact on operations.

If the time frame for the decision to be made is longer, you can use more sophisticated models to solve the problem. Based on the business conditions, the availability of data, the condition of data, the number of objectives of key decision-makers, and the organization's goals, you can combine methods to develop the model.

2.0 Multiple Attribute Decision-Making Problems

Decision-makers often deal with problems that involve multiple, usually conflicting criteria. These problems may involve personal decisions such as what or home to buy or may involve large scale decisions such as what products to produce in a company, which research and development projects to fund, and so on. Within the industry, the government and the military, individuals are making these types of decisions on a daily basis. Multiple Attribute Decision Making refers to making preference decisions over the available alternatives that are characterized by multiple, usually conflicting attributes. Multiple Attribute Decision Making is part of a larger category of problems, Multiple Criteria Decision Making (MCDM), which includes the category of Multiple Objective Decision Making (MODM). The nature of MODM problems differs somewhat from MADM problems in that their form and structure in that MODM problems involve designing the best alternative given a set of conflicting objectives. An MODM problem involves developing a model that has conflicting objectives subject to resource constraints and will allow for an infinite number of design possibilities. This course addresses MADM problems and further addresses the MADM problem where the compensation across attributes are not allowed.

Multiple Attribute Decision-Making models rank alternatives based on several criteria measured with subjective and objective data. These decisions are made with attributes (criteria) and alternatives (choices) that represent the key considerations that are made. One important aspect of these decision-making problems is that both objective and subjective data can be used in solving the problem. Subjective data is typically forward-looking and represents the expert's opinion on the descriptive features of the attribute. Objective data, which is typically

Portions ©2019 Decatur Professional Development, LLC All rights reserved.

historical in nature, is typically the type of data you may find in a corporate database, analytical sources, and raw data information that describes the status of the attribute in relation to the alternative being evaluated.

Other features of the multiple attribute decision-making problem include an attribute hierarchy, conflict among criteria, a hybrid nature, uncertainty, a large-scale problem, an ideal-solution, a non-dominated solution, a satisfying solution, and a preferred solution. For more information in regards to the background and definition of MADM problems, please refer to the course Characteristics of Ranking Problems with Multiple attributes.

2.1 Non-Compensatory and Compensatory Decision-Making Methods

Multiple attribute decision-making problems or problems that rank alternatives fall into two broad categories of methods. The first category is non-compensatory, which means that trade-offs between attributes are not allowed in selecting an alternative. Non-compensatory decision-making models are discussed in more detail in another course developed by this author. The second category is compensatory, where the value of one attribute can compensate for a lower value in another attribute in the alternative selection process. These methods are discussed in more detail in the following sections.

2.2 Compensatory Methods

Compensatory methods permit tradeoffs between attributes. A slight decline in one attribute is acceptable if it is compensated by some enhancement in one or more other attributes. These methods are very useful in real-world decision making. These methods fall into three general categories, with many variations in differentiation between the methods.

- Scoring Methods
- Compromising Methods
- Concordance Methods

Below is a brief definition of these three different types of compensatory methods. In this course, we will be addressing the computations associated with one scoring method with two variations, the Simple Additive Weighting method with both scaling and normalization and the Technique for Order Preference for Similarity to the Ideal Solution (TOPSIS) which is a compromising method. The usefulness of these two methods makes them valuable in private and public sector decision-making problems.

2.2.1 Scoring Methods

The scoring method selects or evaluates an alternative according to its score (or utility). Utility or score is used to express the decision maker's preference. It transforms attribute values into a common preference scale such as [0,1] or [1,5] so that comparisons between different attributes becomes possible. A very popular method in this category is the Simple Additive Weighting method (SAW). This method calculates the overall score of an alternative as the weighted sum of the attribute scores or utilities. The Analytical Hierarchy Process (AHP) is another popular method in this category. This method calculates the scores for each alternative based on pairwise comparisons. Additionally, Multiple Attribute Utility Theory (MAUT) is also a method that would be considered a scoring method.

Portions ©2019 Decatur Professional Development, LLC All rights reserved.

2.2.2 Compromising Methods

The compromising method selects an alternative that is closest to the ideal solution. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method belongs to this category. This method first normalizes the decision matrix of an MCDM problem. Then based on the normalized decision matrix, it calculates the weighted distances of each alternative from an ideal solution and a negative ideal solution. A solution relatively close to the ideal solution and far from the negative ideal solution is evaluated to be the best.

2.2.3 Concordance Methods

The concordance method generates a preference ranking, which best satisfies a given concordance measure. The Linear Assignment Method is one of the examples in this family. In this method, it is believed that an alternative having many highly ranked attributes should be ranked high. ELECTRE is also in this classification of methods.

3.0 The Decision Matrix

This section describes the decision matrix, which is used to structure and solve multiple attribute decision-making problems. The decision matrix provides a structure to capture the key aspects of the decision being considered. It also provides a standard form that can be used to apply different methods to solve the decision problem at hand. The decision matrix can be used to apply both compensatory and non-compensatory MADM methods to arrive at a solution.

3.1 Definition of the Decision Matrix

In general, there exist two distinctive types of MCDM problems. One type of problem has a finite number of alternative solutions and would be considered a multiple attribute decision-making problem and the other an infinite number of solutions, which would be considered a multiple objective decision-making problem. Normally in problems associated with selection and assessment, the number of alternative solutions is limited. In problems related to design, an attribute may take any value in a range. This means that the potential alternative solutions could be infinite. Our focus will be on the problems with a finite number of alternatives.

An MCDM problem may be described using a decision matrix. Suppose there are m alternatives to be assessed based on n attributes. A decision matrix is an m × n matrix with each element a_{ij} being the j-th attribute value of the i-th alternative. Decision criteria or attributes are represented as C_j. Importance weighting associated with the decision criteria is represented at w_j, and the alternatives under consideration are represented by Ai. Below is a representation of the decision matrix and a brief definition of the elements of the matrix.

Weights	**W_1**	**W_2**	**…**	**W_n**
Alternatives/Criteria	**C_1**	**C_2**	**…**	**C_n**
A_1	a_{11}	a_{12}	…	a_{1n}
A_2	a_{21}	a_{22}	…	a_{2n}
…	…	…	…	…
A_m	a_{m1}	a_{m2}		a_{mn}

Portions ©2019 Decatur Professional Development, LLC All rights reserved.

Criteria: A criterion or attribute is a measure of effectiveness. It is the basis for evaluation. Criteria emerge as attributes or objectives in the actual problem setting. In multiple attribute decision making, they can be viewed as performance parameters, components, factors, characteristics, and properties. An attribute should provide a means of evaluating the level of an objective. Each alternative can be characterized by a number of attributes, based on those elements that are important in the decision process.

Weights: Not all attributes are equally important when making a decision. Minimizing cost, for example, might be far more important than maximizing comfort. By assigning weights to the decision criteria or attributes, the decision-maker or team can represent their value of importance in the ultimate decision. As a note, not all decision methodologies support criteria weighting, therefore it is necessary to select a method that is representative of the types of considerations that the decision-maker or makers wish to consider.

Alternatives: Alternatives are those potential choices that a decision-maker can select based on his or her evaluation of the key aspects of the decision. You, for example, might be deciding to purchase a car and use a decision matrix for your evaluation process. Your alternatives are the different types of cars that you are considering, and the decision criteria or attributes are those elements that are important to you in your decision-making process.

Attribute Value for Alternatives: These values are the representation of the performance level for an alternative as it is associated with each attribute. For a given alternative, there is a scoring of each attribute to represent the value and consideration associated with the alternative. These evaluations are the data that are used in the evaluation process.

The decision matrix is the fundamental construct of the multiple attribute decision-making problem and is used as a basis for applying the numerous different types of decision-making methods that can be used to solve multiple attribute decision-making problems. The development of the attributes, alternatives, importance weighting, and data used in the decision matrix is important to developing a well-structured decision problem. Also, the development of the decision matrix is an important activity that can be used to gain consensus from the key decision-makers on the key considerations that are used in the decision process. Group decision-making techniques can be used to support this process. Group decision-making techniques are discussed in detail in another course developed by this author.

3.2 Data Used in the Decision Matrix

You can use either objective or subjective data to represent the metrics used in the decision process. Objective data usually can be described as data that can be quantified by some measure of known commonality. This may be data such as the number of items produced, number of trucks in a location, population of a city, and so on. This data is usually available in some form in company databases and information systems. Typically, statistics such as averages and trends are generated based on a record of this objective data over some period in time. Objective or quantitative data represents a history of activities of a company that has been operating during a given time period.

Qualitative or subjective data can be easily used in a number of different situations. Surveys are good examples of subjective data used to represent the rating of a product or service. Use scales from 1 to 5 or 1 to 10 to represent high, medium, and low assessments for given metrics. Use assessments such as red, yellow, and green in other situations in which individuals (such

Portions ©2019 Decatur Professional Development, LLC All rights reserved.

as military personnel) might find more meaning in rating conditions. Numerical values with their verbal description provide the type of information that can be captured and utilized in a decision model when other information is not available.

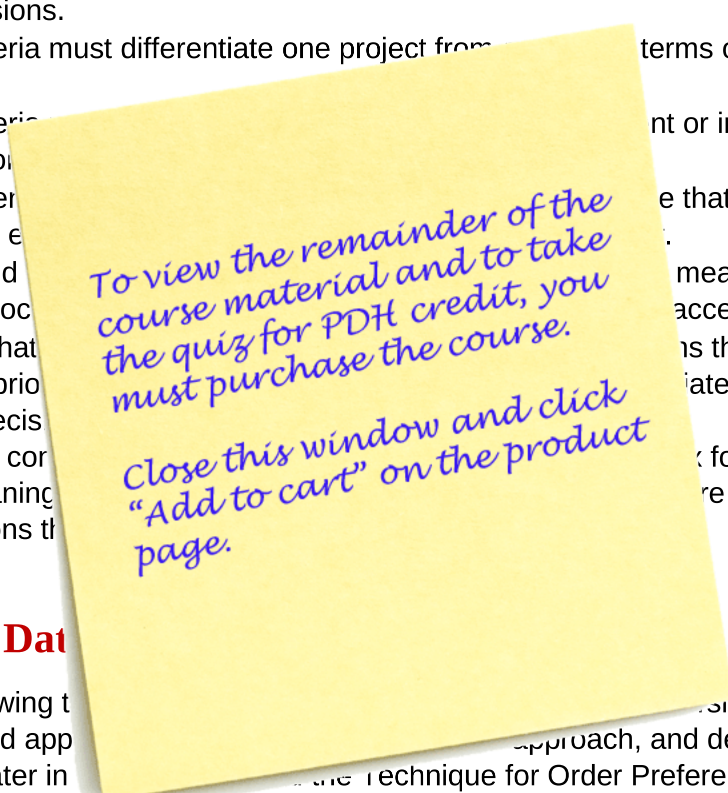
Subjective data is data based on someone's opinion or best guess of a condition or a future event. Subjective data is more qualitative in nature in that it defines a situation or condition without specific data points. Subjective data can be generated by individuals within or outside of a company or experts within a given field of operation. Typically, subjective data or opinions provide insights into a subjective assessment of a metric. Subjective data and expert opinions are typically forward-looking in nature, trying to predict what will happen in the future. Individuals make assessments based on what has happened in the past and what may happen in the future.

Objective data, especially in the form of statistics, however, is based on historical data, thus projecting the future, what has happened in the past, which assumes that the future will behave much like the past. The entire business environment may have changed; thus, what has happened in the past may be a poor representation of the future; thus, a new source of data is required.

In developing decision criteria or attributes that are used in multiple attribute decision-making problems, you should consider a number of parameters in the development process to ensure a set of well-structured, well-represented goals, and decision criteria. These development parameters are as follows:

- Goals and decision criteria must represent actual and important considerations in making decisions.
- Decision criteria must differentiate one project from ... terms of higher or lower priority.
- Decision criteri... nt or intent, to avoid accounting fo...
- Decision criter... e that the decision criteria in the e... .
- Measures and ... meaningful in the evaluation proc... accessible.
- Constraints that ... ns that would be applied to a prio... iated from the evaluation decis...

These are important cor... for decision making. The quality and meaning... re paramount in the quality of the decisions t...



4.0 Example Dat...

The data in the following t... sions of the SAW method, one a scaled app... approach, and demonstrate another method discussed later in ... the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). This table shows a number of new products that are being evaluated by a company to add to its product line. Ten different products (Product A – J) are being

Portions ©2019 Decatur Professional Development, LLC All rights reserved.