Strategic Decision Making with Game Theory

An Online Continuing Education Course for Engineers

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Strategic Decision Making with Game Theory

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1.0 Introduction

This course is an introduction to key concepts in the area of Game Theory and some of the more well-known games that are associated with the field. Game theory is a set of tools used to help analyze situations where an individual's best course of action depends on what others do or are expected to do. Game theory allows us to understand how people act in interconnected situations. Connections between people arise in all sorts of situations. Sometimes we achieve more through cooperation. Other times, conflicts arise, and individuals benefit at others' expense. Many times, there is both cooperation and conflict in a given situation.

Game theory can be very theoretical. However, in this course, we will discuss key topics of game theory and keep the discussion to a practical and applicable level. Engineers should be aware of these strategies when making corporate and even personal decisions. Understanding these concepts and strategies will assist in better anticipating the reactions of other players of the game.

Topics covered in this course include the following:

- Introduction to Game Theory
- Key components of Game Theory
- Simultaneous Move Games
- Payoffs
- Nash Equilibrium
- Best Response Functions
- Prisoner's Dilemma
- Dominate Actions
- Stag Hunt Game
- Free-Rider Game
- Battle of the Sexes
- Zero-Sum Game
- Mixed-Strategy Nash Equilibrium

A two-player example problem is provided with each of the games, as well as a discussion of an area of business application. It is an interesting field with many applications. As the number of players increases in these games, the computational requirements significantly increase. It is interesting, however, to think through the strategic application of these games in business applications.

2.0 Game Theory Terms and Definition

Game theory is a set of tools used to help analyze situations where an individual's best course of action depends on what others do or are expected to do. Game theory allows us to understand and anticipate how others may act in a given situation.

In Game Theory, there are sets of tools and a number of different circumstances where you apply these tools when you are playing some form of a strategic game with other players. The circumstances around the tools dictate the types of actions you might use the actual circumstances. In Game Theory, we look at how we are going to interact strategically with all of the other players in the game and then to select the best response for you based on the circumstances.

These circumstances may involve conflict. In this case, two corporations are trying to gain more market share, and they will do what is necessary to accomplish this goal. Sometimes, there is a game where cooperation occurs, and you formulate this problem and the strategic play differently than a conflict situation. Sometimes you'll take a given action at somebody else's expense, and one party wins it all while the other party loses it all. There is no compromise solution. In this course, we will look at a series of different types of games and assess our best responses to the actual game itself. In economics, you see competition and pricing between firms. In political science, you have different candidates with their own platforms and how voters may interact based on what the candidate presents on their platform. With network computing, you may be looking at the strategic use of network bandwidth and the potential effects if both you and your competitor attempt to utilize all of that bandwidth.

As a side note, there is a movie called "A Beautiful Mind," which stars Russel Crowe. In this movie, Russel Crowe plays the character, John Nash. The concept of a Nash Equilibrium is critical to game theory, so it is worthwhile to watch to see the evolution of John Nash and Nash Equilibrium in the overall context of Game Theory. John Nash won the Nobel prize for his work.

John Nash began developing his theory on Nash equilibrium in a very unusual way. This is based on the story about four guys deciding on which woman they were going to try to talk to for a date. There were four women, a beautiful blonde and three other attractive women. In this case, no one chose the blonde because if she said no, all the other women would think that they were the second choice. So, their best choice was not to choose the blonde and to choose the other women to approach. So, with strategic interaction, how well you do depend on the actions of others playing the game. We need to understand what we know, how to choose the best for ourselves, and what the other player will most likely choose based on the presented payoffs for the presented options and results for the game.

Terminology around game theory comes from the types of terminology that we see in board games. Key terms are shown below:

- ***Players*** are the decision-makers
- ***Moves*** are the decisions that the decision-makers make
- ***Models*** are simplistic structures that capture the important features of the real-world problem
- In game theory, we create simplified ***models*** that we call ***games***.
- Game theory assumes **rationality** and **common knowledge of rationality.**
    - **Rationality** refers to players understanding the setup of the game and exercising the ability to reason.
    - **Common knowledge of rationality** assumes the players of the game are all rational and believe that each will behave rationally in their selections and moves.

3.0 Simultaneous Move Games

A simultaneous move game is a game where it is assumed that all the players make their move at the same time. In this case, sometimes the players are making their choices at the same time; other times, they simply don't know the actions of the other players. This type of game takes on the form of a simultaneous move game. This assumes that we do know all the payoffs of the players in the game, and we can assume that each player will make moves rationally, based on their best response to the actual payoffs given in the payoff matrix.

To construct a simultaneous move game, we will need to know the players, the options that they have for choice, and then the payoff associated with each choice for each player. The steps are described below:

- **Identify the Players.** For a two-person game, one player will list their choices on the rows. The other player will list their choices on the columns. These columns and rows show each players' possible actions.
- **Estimate the Payoffs.** Estimate the payoffs for each player for each of the possible actions.
- **Create a Payoff Matrix.** Create a payoff matrix showing all possible outcomes of the game and what each player will receive as a payoff for those outcomes.

We will use an example of two small, competitive firms launching a similar product. These firms have been in business for some time and have enough information to develop a logical representation of each other's potential launch times and product payoffs. Let's assume that one of the firms is owned by Roger, so we will call it Roger's Firm. The other firm is owned by William, so we will call it William's Firm. Both are looking at launching their product for the Christmas season, so they will be choosing to launch in either July or October. The payoff matrix for this game is shown below. Roger's firm typically has higher quality products than William's firm, so their payoffs tend to be higher. Both firms are aimed at the same demographic. People tend to start thinking more about Christmas in October instead of July, but it would be good to get the product into the marketplace early to test the response. There are different payoffs, depending on when they choose to launch the product. This payoff table details what the payoffs will be for each firm in competition with the other firm, based on when the firms launch the products

	William's Firm		
Roger's Firm		July	October
	July	R: 50, W: 5	R: 70, W: 10
	October	R: 120, W: 7	R: 90, W: 8

To discuss the development of the table further, we see that we will represent each firm's decisions and their choices by rows or columns. For example, Roger's firm's decisions are either to launch in July or October. The rows of July and October represent the two choices that will be evaluated in this game. William's firm's choices are the same, to launch in either July or October. William's firm's decisions will be represented by the columns. We need this payoff matrix to be able to help us decide what we are going to choose to do in this strategic interaction.

If we want to refer to this mathematically, we will call this the strategic form of the game. Each player has a set of actions that July or October launch. Each player has a payoff based on their actions and the actions of the other studios. Looking at the payoffs in the table, we see that the lowest payoffs for both Roger's Firm and William's Firm are if both firms launch their product in July. We see that if William's firm launches in July, Roger's firm will have a higher payoff by launching in October (R:120). If Roger's firm launches in July, William's firm will have the largest payoff if they launch in October (W:10). The payoff matrix is developed based on the specific actions taken by each firm and the resulting actions for that specific strategic interaction. In the next section, we will evaluate the choices in the game. However, we will want to introduce some additional concepts before we do that. This common knowledge is an important concept in game theory. Common knowledge means that each player knows the strategic actions of the other players and their payoff.

4.0 Nash Equilibrium and the Best Response Function

Two key concepts are required to understand and apply concepts in Game Theory. Those concepts are the definition of the Nash Equilibrium and the Best Response Function. Understanding and application of game theory concepts are derived from these concepts and are used to apply these concepts to strategic games played in a variety of business, environmental, and personal environments.

Nash Equilibrium

The Nash Equilibrium, named after the American mathematician John Nash (1928-2015), is the fundamental concept in game theory. Nash did not invent the idea, but applied it to the mathematical analysis of games in general. In equilibrium, each rational player chooses his or her best response to the choice of the other player. This means he chooses the best action given what the other player is doing.

The action profile in a strategic game with ordinal preferences is a Nash equilibrium if, for every player and every action of the player, their choice is at least as good according to the player's preference as the action profile in which every player chooses their best set of actions.

Games may have a single Nash equilibrium, no Nash equilibrium, or many Nash equilibria. The assumption is that this results in a steady-state among experienced players where they will choose their best action profile in reference to the other players' best action profile for the game. This model is a steady-state among experienced players.

Best Response Function

To determine the Nash Equilibrium, we must determine the best response function for the game. This is determined by noting each player's best interaction, given the choice of another player. This action must be at least as good as every other action in the choice set.

Basically, the best response set for a given game or payoff table is the one where the player makes the most logical choice, given the choice of the other player. Because we know the payoffs, Player 1, for example, knows the choices of Player 2 and his own payoffs associated with those choices given the game and play scenario.

The method for finding the best response function for finding the Nash equilibria is discussed below. However, we identify what we need in terms of the best response function for each player.

- Find the best response function of each player
- Find the action profiles that satisfy each player's single best response to the list of the other players' actions

Finding a Nash Equilibrium

As mentioned before, we need to find the best response function for every player before we can find the Nash Equilibrium. To do so, we must calculate the best response functions of every player. Once that is done, we need to check which outcomes (if any) lie on the best response function of every player.

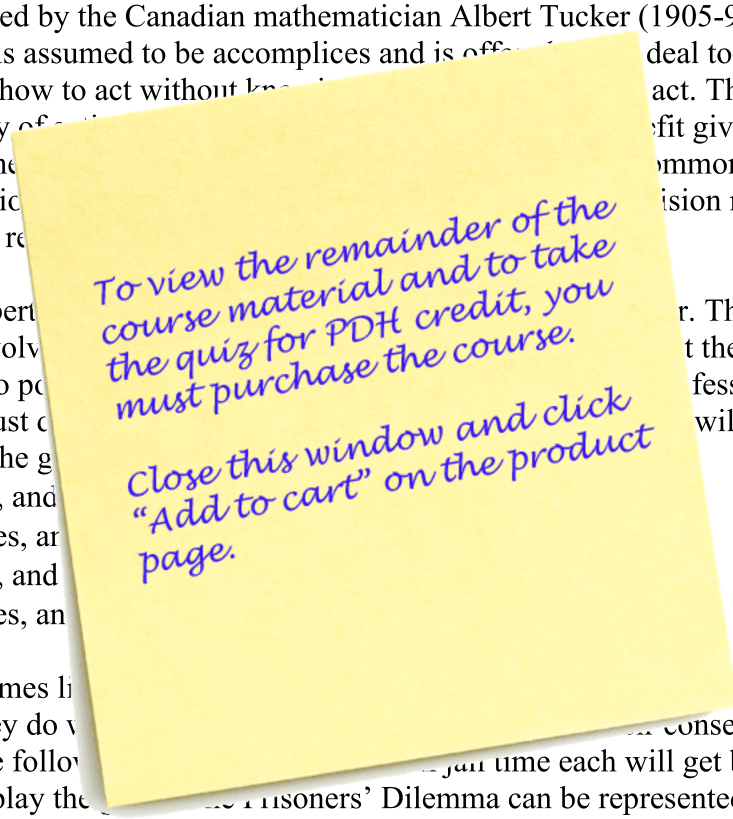
Let's walk through the product launch problem between Roger's Firm and William's Firm to determine the Nash Equilibrium for the problem and the payoff matrix we developed above. If William's Firm decides to launch the product in July, Roger's firm will need to then choose which payoff is bigger for its product launch, which is R:50 for July and R:120 for October. Roger's firm would choose October with a payoff of R:120 since it is the bigger payoff. If William's Firm chooses to launch in October, Roger's firm's largest payoff is R: 90 for October, so Roger will choose to Launch in October. If Roger's firm chooses to launch in July, William's Firm will have the largest payoff if it launches in October, where W:10. If Roger's Firm chooses to launch in October, William's firm will have the largest payoff if it launches in October W:8. The best response function for each of the firms has been underlined in the table below.

Roger's Firm	William's Firm		
		July	October
	July	R:50, W:5	R:70, W:10
	October	R:120, W:7	R:90, W:8

So, what is the Nash equilibrium? The Nash Equilibrium is the best response function for both players. In this case, it will be for both firms to launch in October. In October, neither firm will deviate from its choice, based on the strategic interaction and payoffs from the game. For example, if William's firm deviated from its October choice and chose July, Roger's firm would still choose October with a larger payoff, and then a smaller payoff would result for William's firm. If Roger's firm deviated from its decision and chose July instead of October, it would result in a smaller payoff with William's firm achieving a larger payoff. Hence, no firm would deviate from the best response function and not cause worse outcomes than what is realized with the best response function. So, one cannot look only at the highest payoffs in the matrix, and the decisions should be based on the knowledge of all of the information in the matrix, the best decisions that all of the players make at that point in the team. The Nash equilibrium is regret-free. No firm would benefit if it were to deviate from its equilibrium strategy, which is why the term equilibrium is important.

5.0 Prisoner's Dilemma

The prisoner's dilemma is one of the problems that is commonly mentioned in game theory. The dilemma was nicknamed by the Canadian mathematician Albert Tucker (1905-95). In this case, each of two prisoners is assumed to be accomplices and is off... deal to confess. The prisoners must decide how to act without kn... act. This problem illustrates the difficulty of ... fit given that people pursue self-interest. The ... mmon problems in areas such as competitio... ision making in animal competition for scarce re...



In this illustration, Albert ... r. The police suspect that they were also involv... t them for it. Albert and Bob each have two po... fess. They are put in separate rooms and must d... will do. The four possible outcomes of the g...

- Albert is silent, and ...
- Albert confesses, ar...
- Albert is silent, and ...
- Albert confesses, an...

Each of the four outcomes li... n this case, depending on what they do ... consequences are based on jail time. The follo... jail time each will get based on their decision as to how to play the ... Prisoners' Dilemma can be represented in strategic form: