Modal Analysis in Excel

An Online Continuing Education Course for Engineers

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Introduction To Modal Analysis

In engineering analysis of mechanical systems and civil structures, the fundamental characteristics of loading generally fall into two categories to consider for design; static loads and dynamic loads.

Static Loads: These are considered constant, and theoretically time-invariant. The net responses to static loading in a structure are constant strain conditions and static equilibrium.

Dynamic Loads: These are time-dependent loads. The net response of a system or structure subjected to dynamic loads generally does not result in a static equilibrium condition.

It is important to mention that under certain conditions, some design standards or specifications provide a designer with the option to represent dynamic loads as static loads as a matter of convenience. As an example, ASCE/SEI 7-16 [1] permits the use of an equivalent lateral force procedure for a structure undergoing earthquake loading in lieu of conducting a dynamic analysis, provided the structure meets certain criteria. If a structure is too flexible or has too many irregularities, the equivalent lateral force procedure is not permitted for use. In this case, the designer must choose an alternative analysis procedure directly utilizing the structure's mass and stiffness characteristics, often involving its natural frequencies and modes.

Load Classification	**Characteristic**	**Examples of Sources of Loading**
Static Loads	Time-Invariant	Dead weight, hydrostatic loads, snow loads, certain transient loads (i.e., traffic loads on bridge, live loads in buildings, loads due to thermal effects)
Dynamic Loads	Time-Dependent	Random vibrations, wind, reciprocating machinery, earthquakes

With static loads, one can readily determine the solution to determine a system's response with internal forces, displacements, and reactions in a single-step calculation provided the overall stiffness is known. With dynamic loads, determining a system's response generally requires a higher degree of computational effort and may involve a series of calculations. In addition to the system's mass and stiffness, any damping must be considered as well. The loading action may not take the form of any particular mathematical function of time but may consist of discrete force or acceleration data points recorded over a specified time interval. For multiple degrees of freedom systems undergoing dynamic loading, a commonly applied intermediate step to find a dynamic response involves *modal analysis*. Although not an absolute requirement for all dynamic analysis problems, modal analysis is widely regarded as the most powerful tool to solve dynamics problems. This course focuses specifically on classical modal analysis procedures.

What exactly is a modal analysis? How does it differ from the dynamic analysis? Why is modal analysis important?

Modal analysis is an investigation of how a system moves when no external loading is acting. It reveals a system's set of natural frequencies, natural modes of motion or vibration, and how much modal mass is captured for each mode. These characteristics can be revealed upon introducing some strain or kinetic energy into a system and analyzing its response.

Suppose that a series of recorded earthquake motions historically have measured peak horizontal ground accelerations occurring at or near a frequency of 2 Hz in a seismically active location. An engineer designing a bridge or building for this location must take proper measures to avoid the structure's fundamental frequency falling near 2 Hz in order to avoid resonance. Modal analysis can dictate where the natural frequencies of a structure will be situated, and the total mass and stiffness can be altered accordingly during the design phase to reduce the risk of harm to occupants. If for whatever reason it is cost-prohibitive or otherwise difficult to avoid a fundamental frequency of near 2 Hz by changing the mass or stiffness, modal analysis can influence decision-making for other measures to be taken such as having the structure equipped with a high degree of ductility, special dampers, or introducing a base isolation system.

Sometimes the terms *modal analysis* and *dynamic analysis* are used interchangeably, but these are not the same.

For the purposes of discussion in this course, a dynamic analysis encompasses the complete response history of a structure subjected to dynamic loads (i.e., time-dependent deformations, internal forces, and reactions). Modal analysis is an *analytical method* used to arrive at a completed dynamic analysis, but in itself does not necessarily represent nor reflect real loading conditions.

Despite this, modal analysis is usually a crucial step in solving dynamics problems because it can seamlessly piece together information to form the basis of a dynamic analysis solution.

This course provides a brief review of the theory behind modal analysis for single degree of freedom and multiple degrees of freedom systems prior to covering worked examples.

Modeling Idealizations

This course covers systems with discretized stiffness and lumped masses. All systems are linear.

Single Degree Of Freedom Systems (SDOF)

Before entering into the discussion of applied modal analysis, a single degree of freedom (SDOF) system is introduced. In an SDOF system, a dynamic analysis considers the interplay of three energy sources:

- **Mass**: Absorbs kinetic energy
- **Spring**: Absorbs strain energy
- **Damper**: Dissipates energy as mass travels relative to the wall (see the following figure)

In the following figure, a simple SDOF system is shown with the mass block m sliding on a frictionless surface; also shown is spring element k and linear viscous damper element c. Also shown is a wall, fixed in place which serves as a support or restraint for the system. The spring and damper elements connect the fixed wall to the mass block. The displacement or deformation $x(t)$ is measured relative to the position of the block where the spring and damper elements are undeformed.

Because this undeformed position, measured *s* from the wall as shown is always a fixed distance, the velocity and accelerations of distance *s* are always zero, relative to the wall. Therefore, in this case, the rate of deformation or velocity of the block $x(\dot{t})$ and the block acceleration $x(\ddot{t})$ can also be measured relative to the fixed wall as a convenient reference point.

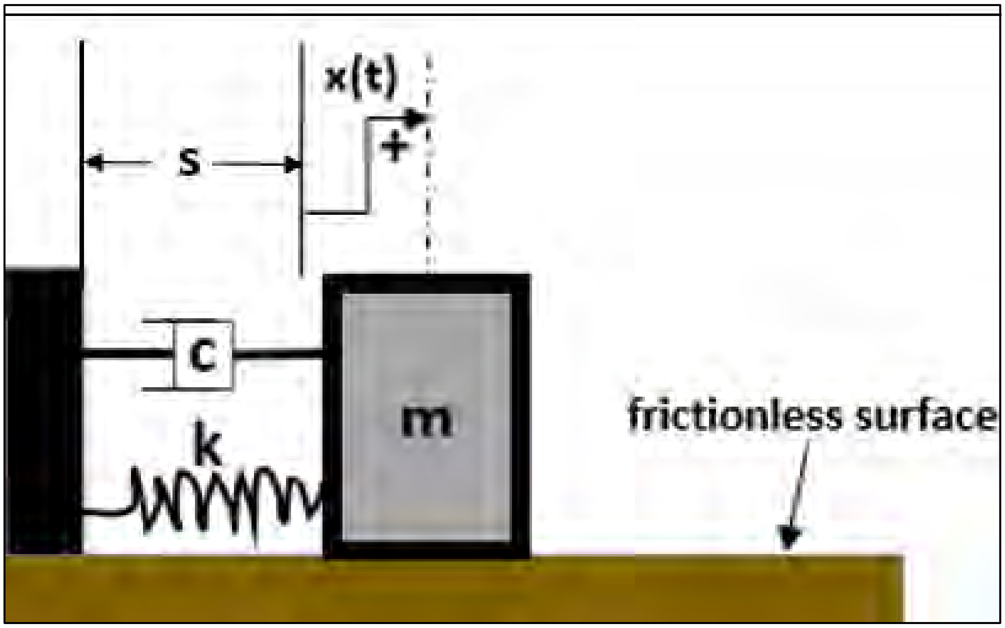


Figure: Simple SDOF System

With initial conditions of nonzero displacement (deformation) of the spring and/or velocity of the mass at time zero established, the equation of motion is shown for an SDOF with no external forcing:

$$mx(\ddot{t}) + cx(\dot{t}) + kx(t) = 0$$

Where:
m = mass
c = linear viscous damping constant
k = spring constant
$x(t) = time\ dependent\ deformation\ of\ spring\ or\ damper$
$x(\dot{t}) = time\ dependent\ velocity\ of\ mass, relative\ to\ wall\ support$
$x(\ddot{t}) = time\ dependent\ acceleration\ of\ mass, relative\ to\ wall\ support$

In our discussion of classical modal analysis, the damping term c does not enter into the scope because the classical modal analysis is limited to undamped free response only. Therefore, the equation of motion simplifies to:

$$m\ddot{x}(t) + kx(t) = 0$$

With the initial conditions x_0 and $\dot{x}_0$ known, a solution to $x(t)$ can be found:

$$x(t) = x_0 \cos(\omega t) + \frac{\dot{x}_0}{\omega} \sin(\omega t)$$

where:

$$\omega = \sqrt{\frac{k}{m}} \quad (natural\ frequency\ of\ system)$$

In this system, there is simply an exchange of strain and kinetic energy with no energy dissipated. The solution is completely characterized by a harmonically varying amplitude of displacement equal to:

$$X = \sqrt{{x_0}^2 + \left(\frac{\dot{x}_0}{\omega}\right)^2}$$

With natural frequency:

$$\omega = \sqrt{\frac{k}{m}}$$

The equation of motion can be simplified and rewritten in terms of simple amplitude X and characteristic natural frequency ω.

$$x(t) = X \cos(\omega t - \alpha)$$

where:

$$\alpha = \tan^{-1}\left(\frac{\dot{x}_0}{\omega x_0}\right)$$

The Generalized Eigenvalue Problem

In most structural analysis problems involving multiple degrees of freedom, the global stiffness matrix ***k*** represents the overall structural stiffness; this matrix is square and symmetric.

For statics problems, the displacement vector ***D*** and static force vector ***F*** are now introduced**.** The solution we seek is the global static displacement of each degree of freedom in the structure. The solution is computed in a single step, and in essence, the computational effort required to complete the analysis is summed up in this single-step calculation:

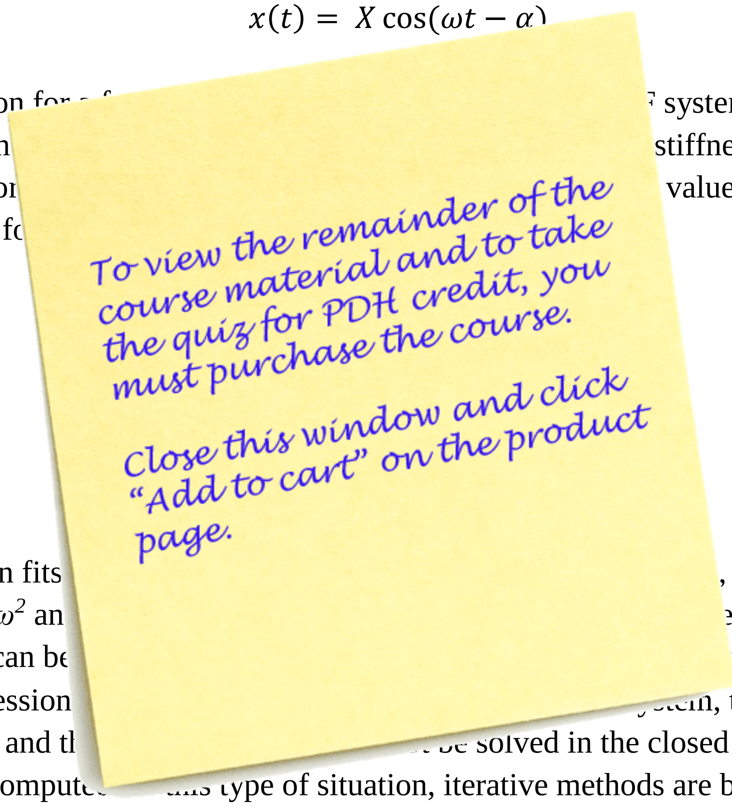
$$\boldsymbol{D} = \boldsymbol{k}^{-1}\boldsymbol{F}$$

For dynamic analysis of MDOF systems, the above equation and solution are insufficient because the aforementioned system is in equilibrium and violates the very nature of dynamics. In addition, the stiffness matrix ***k*** is not always invertible for dynamics problems as will be demonstrated later.

In dynamics problems, the global mass matrix **m** enters into the picture as well; this matrix is also square, symmetric, and oftentimes diagonal.
A convenient starting point for a general solution utilizes the general solution of an undamped SDOF of freedom oscillating system defined earlier:

$$x(t) = X\cos(\omega t - \alpha)$$

The SDOF solution for a f … F system. Foran MDOF system, the ***x(t)*** an … stiffness matrices as well as an unknown vector … values back into the SDF system yields the fo…



The above relation fits … , with a set of eigenvalue roots ω^2 an… …ee of freedom system, the two ω^2 roots can be… …n from the determinant expression… …stem, the calculation process is more involved, and th… …be solved in the closed form. When many ω^2 roots need to be compute… …this type of situation, iterative methods are best suited to solve the