Helical Gear Design, Analysis, and Applications

An Online Continuing Education Course for Engineers

Course Number: M-2098

Credit: 2 Hours / 2 PDH / 2 CPD

Helical Gear Design, Analysis, and Applications

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1. Objectives

- Familiarity with different types of Helical Gears
- Advantages and Disadvantages of Helical Gears
- Common helical Gear Nomenclatures for Design Calculations
- Gear Classifications and Helical Gear Design Considerations
- Continuous Helical Gear, Herringbone Gear
- Crossed Helical Gears
- Helical Gear Material and Heat Treatment process
- Design Strength and Wear Rate of Helical Gears
- Design Horsepower Calculation for helical Gears
- Design Horsepower for Helical Gears
- Gear Tooth Failure Mechanism
- Gear Force and Stress Analysis
- Gear Shaft and Support Bearing Analysis
- Cause and Mitigation of Gear Noise

2. Introduction

This Course is primarily devoted to the design of helical gears and their analysis and properties. Any gear has to resist bending stresses during load transmission. Teeth fail due to such bending loads. Pitting and premature wear occur due to excessive loads and speeds. Failure of the teeth due to stress occurs when bending stress goes beyond the material's allowable stresses, and surface failure occurs when contact stress at the contact point exceeds the endurance limit of tooth material. Lubrication of teeth is also very important and significant to enhance gear life.

Proper use, strength calculations, Prevention of excessive wear and pitting failures of gear teeth, gear materials, noise mitigation, lubrication methods, etc., have been recommended by the American Gear Manufacturers Association (AGMA) for a long time in the United States. Whenever required, AGMA practices have been mentioned in this chapter also. This chapter is a simplification of AGMA recommended procedures, but important procedures have been simplified for practicing design engineers. The focus of this chapter has been to explain the behaviour and application of helical gears.

It is hard to imagine a production machinery without having a set of gears in it. Gear is a basic mechanical element for speed or torque management. This is also a fact that even today, designers and manufacturers are constantly working to improve the gear design and manufacturing process to produce the best set of gears for various applications throughout the airline, marine applications, earth moving equipment, and machine tool industries. The gear design started way back right from the Renaissance time and it is still going on as we speak.

At this point, several types of gears and gear arrangements have been used in machine design. The author has also been associated with several such gearbox designs using spur, helical, bevel, and worm gears throughout his professional career over fifty years. The present chapter will reflect some of those experiences to help future designers design helical gears. The author, during his time, always depended on the feedback and help from several gear manufacturers such as Philadelphia Gear Corporation, Arrow Gear Corporations, etc. to name a few. The contents of this chapter are very much guided by the inputs received from these two companies with comments from the author.

The contents have been put together to focus on two aspects: Helical Gear design principles and applications to be applied for machine tool industries and automobile applications. The chapter will contain force analysis, durability analysis, noise reduction for gear applications and will also contain the methods to design gears and gearboxes for and durability analysis techniques. The principles of gear nomenclatures and manufacturing will be covered in brief since designers are not required to know that as gear manufacturers will help the designers to that extent.

The simple shortcut formulas for designing gears of various types will be demonstrated in this chapter. It will demonstrate the methods and processes that the author used to design gears and gearboxes for machine tool applications only. For various applications, durability and strength horsepower of helical gear sets, noise reduction, etc, are very pertinent requirements that designers must satisfy. Helical gear design principles will be explained very briefly for a typical torque reducer that the author has applied in the indexing unit of a CNC turning center. Many of the contributors are very well-known in the gear industry, such as Darle Dudley, Elliott Buckingham, Gene Shipley, Prof. M F Spotts, and many others.

The contents should help the designers to a great extent in applying the design principles for their design efforts. Details of gear classifications as per the American Gear Manufacturers Association (AGMA) will be briefly explained as and when necessary. In general, the designers need to know the design approach and design process for gears for machine tools and marine and automobile applications where durability, cost, and noise reduction are very important design considerations. Applications, examples, and design discussions will be limited to helical gears that are mostly applied in machine tool applications. For applications such as heavy earth moving equipment, ship engines, automobile transmission, etc. helical gears are found to be very suitable and cost-effective.

In this chapter, we will learn about the following aspects of gear design applications:

- The strength Horsepower of helical gear sets
- Durability Horsepower of helical gear sets
- Helical Gear Set Design Principles
- Helical Gear Teeth Forces
- Design strength and Wear rate considerations
- Gear Noise Reductions

3. Advantages and Disadvantages of Helical Gear

Both spur and helical gears transmit power between parallel shafts, but helical gears have specific advantages over spur gears. Helical gears have higher load-carrying capacity than spur gears since more than one tooth carries the load, i.e., the contact ratio is more than one for helical gears. Helical gear teeth are not parallel to the axis of the gear as in spur gearing. The teeth are cut at an angle to the axis of rotation, called the helix angle of the teeth. The mating gear must have the same helix angle, but they are cut in opposite hands, i.e., Right-hand teeth engage with Left-hand teeth or vice versa. The teeth have opposite hands.

As mentioned before, helical gears have more than one contact ratio, i.e., more than one tooth shares the load. Before one tooth leaves the contact, another tooth comes in contact at the pitch line. This is called overlapping of the teeth, which also means that one tooth is always in contact during load transmission. Due to this property of helical gearing, higher pitch line velocity is possible than spur gearing. Due to the higher than one contact ratio, helical gears run much smoother with less noise and vibration than those for spur gearing. The overlapping is optimum when the helix angle is between 7 degrees and 23 degrees. Due to the helix angle, the single helix helical gears produce axial thrust parallel to the axis of rotation, and the support bearings have to be designed to take care of this axial thrust. This axial thrust is in addition to radial and tangential loading on the teeth. Spur gear has zero helix angle, so it does not create any axial thrust. In principle, angular contact bearings or Taper roller bearings are used for helical gearing to resist this axial thrust. This axial thrust is a function of helix angle, i.e. higher the helix angle, the greater the axial force produced.

Due to the helix angle of the teeth, tooth nomenclatures and pressure angle are defined in two different ways. When “normal” is added to the circular pitch, diametral pitch pressure angle, etc., they are measured along the axis perpendicular to the tooth axis. The normal values are used to calculate the tooth strength also. As opposed to the “normal” designations, “Linear” measurements of pitch circle diameter linear, diametral pitch linear, linear pressure angle, etc., are done in the plane of rotation. This will be explained below.

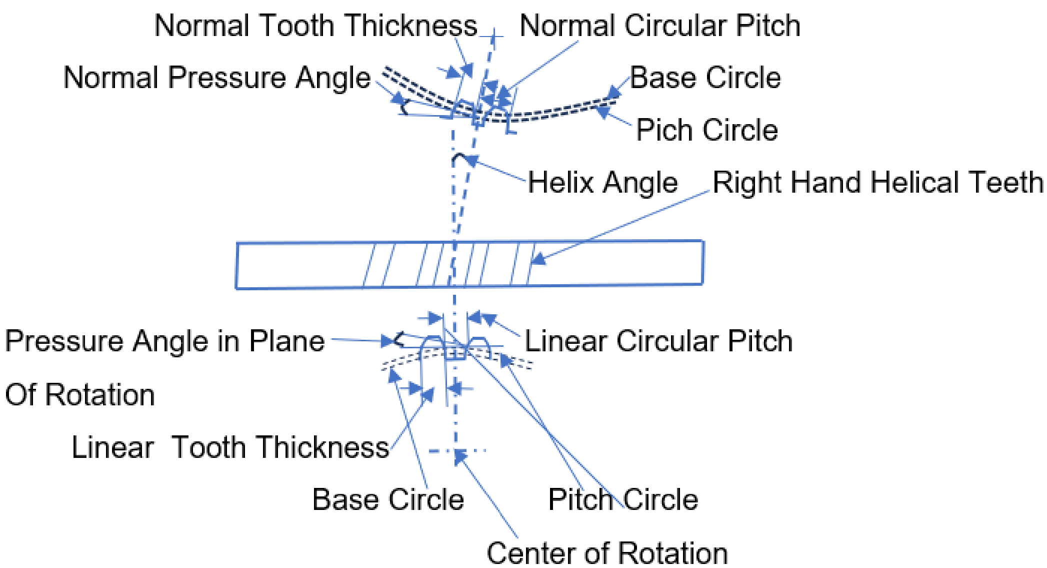

Fig.1. Helical Gearing Nomenclatures (Source: Philadelphia Gear Corporation Handbook)

<u>Helical Gear Nomenclatures:</u>

(Source: Philadelphia Gear Corporation Handbook)

N_G = Number of Teeth in Gear; N_P = Number of Teeth in Pinion

D_G = Pitch Diameter of Gear; D_P = Pitch Diameter of Pinion

D_O = Outside Diameter of Gear; C Center Distance between Pinion and Gear

P_d = Linear Diametral Pitch; P_n = Normal Diametral Pitch

p_d = Linear Circular Pitch; p_n = Normal Circular Pitch

h_t= Whole Depth;

$\emptyset$= Linear Pressure Angle; $\emptyset_n$= Normal Pressure Angle

Ψ= Helix Angle;

t_n = Normal Teeth thickness at pitch line; t = Linear Teeth thickness at pitch line

M_G= Gear Ratio

Formulas for Helical Gearing as per AGMA:

(Proportion based on Normal Pitch; Source: Philadelphia Gear Corporation Handbook)

P_n = Normal Diametral Pitch = N/(D*CosΨ); Outside Diameter= D + 2a

p_n = Normal Circular Pitch = 3.1416/P_n ; t_n = 1.571/P_n; t = 1.571/(P_n CosΨ)

D = Pitch Diameter = N/(P_n*CosΨ)

C = Center Distance = (D_P+ D_G)/2

a = Addendum of Teeth = 1/P_n; d = dedendum of Teeth = 1.157/P_n

h_t = Whole Depth of teeth = 2.157/P_n ; M_g = Gear ratio = D_G/D_P

4. Gear Nomenclatures

Some of the basic but important nomenclatures/terminologies for helical gears are as follows:

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p_d = Linear Circular Pitch; p_n = Normal Circular Pitch

h_t= Whole Depth;

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C = Center Distance = (D_P+ D_G)/2

a = Addendum of Teeth = 1/P_n; d = dedendum of Teeth = 1.157/P_n

h_t = Whole Depth of teeth = 2.157/P_n ; M_g = Gear ratio = D_G/D_P

5. Types of Machine Tool and Automobile Gears

In general, some of the types of gears used in machine tool applications are mentioned below:

- Bevel Gears
- Spiral Bevel Gear
- Zerol Gears
- Helical Gear
- Herringbone Gear
- Crossed Helical Gear

In machine tool applications, Philadelphia Gear Corporation has facilities and capacities for manufacturing helical gears. External Helical gears with a maximum pitch diameter of up to 300 inches can be manufactured in the facility with a maximum Outside diameter of 302 inches. Maximum face width is 59-60 inches.

6. AGMA Gear Classifications

The gear classifications are established by Gear Manufacturing Associations (AGMA) as per standard 390.02. It specifies the selection and specification of gears for industrial uses and applications. Gears are assigned AGMA numbers, which combine quality number and heat treatment process applied to manufacture the gears. For example, a gear specified by number 9-H-16 means gear quality number 9, with hardness 335 to 375 BHN.

AGMA quality number ranges from 3 to 15. The higher the number more precise the gear is. For indexing purposes, I always used quality number 10 to control the accuracy of indexing the turret. For Gear Box applications, class 9 is used to reduce the sound and accuracy of running. For higher speed, gear class 9 or better must be used to control runout tolerances, which is twice the eccentricity of the gear pitch circle.

Other qualities of the gears include tooth-to-tooth spacing tolerance, profile tolerance, total composite tolerance, lead tolerance, etc., which are given in detail in the AGMA standard 390.02.

7. Helical Gear Design Considerations

Helical Gears transmit power between two parallel shafts. The gear set transmits power, creating axial thrust on the support bearings mounted on the shaft. Only radial loads are created. Ground and heat-treated gears can be used for high speeds and power transmission without much problem.

Helical gear material could be cast steel, alloy steel, carbon steel, and phenolic resins. Materials must be selected to have enough durability and strength for the gears for the application. Helical gears can be hardened to a specified hardness number using oil or water quenching, carburizing nitriding, and induction hardening processes.

8. Helical Gear Materials and Heat Treatment

The selection of materials will depend on the strength and durability requirements of the gear system. The material could be cast iron, cast steel, carbon and alloy steel, plastics, etc. Steel materials could be thoroughly hardened, or induction hardened depending on the requirements. The steel hardening process includes oil or water quenching. Gear teeth can also be case carburized and normalized to add higher toughness to the teeth. The author has used SAE 1020, Nitride Steel, SAE 1040, Class 30 or 40 Cast Iron for spur gear very successfully in machine tool applications. The allowable contact stress for the gear/pinion depends on the type of material used and the heat treatment applied.

Allowable bending Stress of the material is also required to calculate the horsepower rating of the gear and pinion set. Some typical values are shown below:

Type of material	Hardness value	Contact Stress, PSI, S_{ac}
Thorough Hardened Steel	300 BHN	120-135,000
Thorough Hardened Steel	440 BHN	170-190,000
Case Carburized Steel	55 R_c	180-200,000
Flame or Induction Hardened Steel	50 R_c	170-190,000
Cast Iron, AGMA Class 30	175 BHN	65-75,000
Cast Iron, AGMA Class 40	200 BHN	75-85,000
Tin Bronze, AGMA 2C	Tensile Strength=40,000 psi	30000

(Source: Philadelphia Gear Corporation Handbook)

Fig.1A: Typical Gear Material Contact Stress and Hardness values

For machine tools, marine, earthmoving, or automobile applications, gear surface finish is very important and should be understood properly. The surface finish is achieved by the grinding process during manufacturing after hardening is completed. Several errors, such as pitch error, lead error, etc., can be controlled precisely by the accurate grinding process. Higher speeds of rotation, vibration effects, and noise reduction for gears can be controlled by dynamic balancing of gears.

Almost all the gears in most of the applications are heat-treated to increase load-carrying capacity. Gear hardening is obtained either by the surface hardening process or through the hardening process. Thru hardening of steel is done using oil quenching to a specified hardness.

Hardening can also be done using carburizing or nitriding process. The surface hardening is done to enhance the wear rate of the gears. Next is the induction hardening process using electric coils. It is a very effective process but it is a very costly process compared to another hardening process. Flame hardening is also used to harden the gears depending on the gear materials. (See Fig.1 also for helical tooth nomenclatures)

9. Double Helical Gears

Double helical gears have two sets of opposed helical teeth in the same blank. Both of these teeth must have the same tooth nomenclatures, i.e., helix angle, pitch, Diametral pitch, etc. The only difference is that the helix angles for the teeth are opposite to each other. This type of gear has a much higher load capacity and durability life. In a double helical gear, axial forces created by each helix are opposite to each other hence, there is no axial force created by double helical gear. Hence, thrust loading created by single helical gear is eliminated for double helical gear. This is a very special advantage for the double helical gear (See Fig. 2 below).

As mentioned before, if the helix angle for the single helical gear increases, axial thrust is also increased. For double helical gear, a higher helix angle is allowed since there is no resultant axial thrust transmitted to the shaft. A larger helix angle also increases the contact ratio and a higher contact ratio helps to run the gears much smoother with higher load-carrying capacity. Double helical gears also have a higher durability rating.

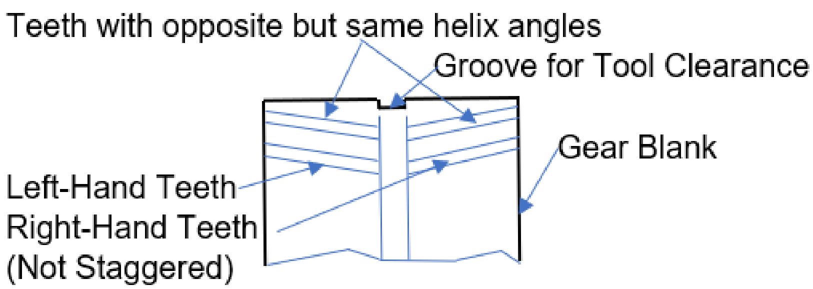


Fig.2: Double Helical Gear Teeth Arrangement (not staggered)

These gears are cut using the hobbing process. To have a double helical set of teeth, a groove in between the set of teeth is required to allow for tool clearances during the hobbing process. The groove width between two sets of teeth must be kept as small as possible to increase the face width of the teeth within a given blank width. In some applications, for staggered teeth arrangement, one set of teeth is in advance of the other set of teeth. This set of tooth arrangements allows the groove width to be smaller than the case when both sets of teeth are aligned.

Moreover, staggered teeth allowed much smoother teeth engagement action since tooth engagements do not occur same time. The staggered teeth double helical gear with a higher helix angle close to 20 degrees can be used for very high pitch line velocity, less noise, no axial thrust, and high durability strength. To achieve high velocity, heat treatment of materials, gear shaving, and grinding must be done. Generally, the heat treatment procedure for helical gears is the same as that for spur gears. Helical gears are treated with flame hardening or case hardening process for smoother operation and higher teeth durability strength. Double helical gearing with hardened and ground teeth will provide optimum operating conditions. For higher speeds, the gear has to be dynamically balanced to reduce noise and shaft whipping. When properly designed, double helical gears can be used up to 20,000 rpm or higher or up to 20,000 ft/min pitch line velocity.

The nomenclatures for double helical gears are given in the AGMA Handbook. These nomenclatures also depend on the manufacturers who hob the gears. For example, as per the Philadelphia Gear Corporation recommendation, the addendum of the tooth, a, can be found out by dividing 0.8 by the diametral pitch of the gear i.e.

$0.8/P_d$. Similarly, the dedendum is given as the inverse of the diametral pitch, i.e., $1/P_{d,}$ and the whole depth of the tooth is equal to $1.8/P_{d,}$ i.e. whole depth is equal to the addendum + dedendum of the tooth. These dimensions differ from single helix gears.

The dimensions of groove length and depth also depend on the tool and manufacturing process used to cut the gear. For example, for teeth in line, the groove dimension is ¾ inch for 16-diametral pitch teeth. The grove dimension increases as the diametral pitch is reduced since gear teeth get thicker and the tool needs more clearance. For example, for a 6 DP tooth linear pitch double helical gear, the groove length and depth are 1.25 inch and 3/32 inch, respectively recommended by Philadelphia Gear Corporation. For staggered tooth gear, the groove width is much smaller for tool clearance. These recommended values are all for 23-degree helix angle gears.

The dimensions and nomenclatures for double helical gears are the same as those for single helical gears. The Helix angle could go as high as 45 degrees to provide maximum tooth clearance. Generally, for a double helix angle hobbed gear, the helix angle for a stub tooth is less than 23 degrees with a 20-degree linear pressure angle. If the helical needs grinding of the tooth, the groove clearance is much larger to accommodate the grinding wheel. For example, For example, for a 6 DP tooth linear pitch double helical gear grinding, the groove length and depth of 1.25 inches and 2 and 15/16 inches, respectively recommended by Philadelphia Gear Corporation.

10. Continuous Tooth Herringbone Gears

For continuous herringbone gears, there is no groove between the two sets of teeth. As shown below, continuous tooth helical gear (herringbone gear) is a double teeth helical gear without a groove in between the set of teeth. The gear manufacturing process is developed by Sykes, and these gears have a much higher load-carrying capacity. These gears are also called “gears with a backbone.” Such gears are used for power transmission at moderately low speeds and high torque. Such gears also provide very little noise power transmission where continuous service is required. These gears are capable of transmitting higher torque up to 3500 rpm and 4000 ft/min pitch line velocity. Such gears have a higher contact ratio and active tooth face width than the double helical gears.

Moreover, it does not create any axial thrust, but a much higher helix angle is possible. Gears can be cut with a 30 30-degree helix angle with a linear tooth pressure angle equal to 20 degrees. (See Fig.3). Because of the absence of a groove, a much higher speed than 3500 RPM is not recommended for herringbone gears, and double helical gears with grooves should be used. Herringbone gears can be cut with a maximum pitch diameter of 60" and minimum pitch diameter of 1 inch, as per Philadelphia Gear Corporation. Gear materials are the same as spur gears. In addition to regular heat treatment procedures such as induction hardening, case carburizing, quenching, etc., a special flame hardening procedure for herringbone gears has been developed by Philadelphia engineers to avoid teeth distortion. Such a method also increases the durability capacity of the continuous teeth gears.

Continuous teeth gears cannot be shaved but can be lapped to enhance the surface finish of the teeth. Lapping reduces the noise and increases the tooth contact ratio.

Continuous tooth herringbone gear is manufactured as per Sykes method with a helix angle of 30 degrees and linear pressure angle of 20 degrees. Different manufacturers might have other possibilities. For continuous teeth herringbone gears, the addendum for the driver is different than that of the driven gears, depending on the ratio of the transmission. This is done to avoid undercutting of the teeth which weakens the teeth during continuous use. For a ratio of 3:1 or higher, for 6 DP teeth, add 0.4 to the pitch diameter of the driver and 0.133 to the pitch diameter of the driven gear. For a ratio less than 3:1, for 6 DP teeth, add 0.267 to the pitch diameter of the driver and 0.267 to the pitch diameter of the driven gear. The groove depth and diameter are also different from standard double teeth helical gear. Consult the manufacturer for further details.

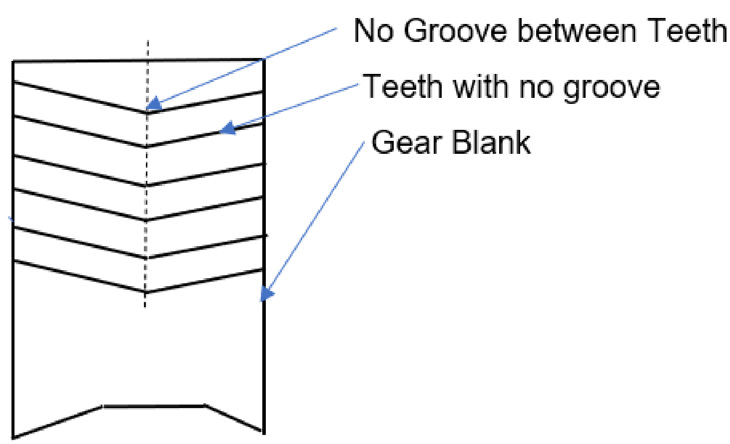


Fig.3: Herringbone Gears, Continuous Teeth

11. Horsepower Rating of Helical Gears

Philadelphia Gears recommends following sequential steps to determine the horsepower rating of the spur gears:

- Determine the strength horsepower Rating of the set.
- Determine the contact ratio or overlap ratio of teeth.
- Calculate the Durability horsepower rating of the gear or pinion.

- Apply service factor for the application using several application considerations on the Durability and Strength horsepower rating of the gear set.
- Divide the durability and strength of horsepower by the service factor.
- The lower of the two-horsepower ratings (Durability and strength) is the horsepower rating of the set, which must be higher than the required horsepower to be transmitted.

Strength horsepower dictates premature failure of the gear tooth due to power and speed transmission. It dictates the weakness of the gear tooth against power transmission. On the other hand, durability strength dictates the wear rating of the gear during power or speed transmission. As per AGMA standard recommended procedure, the strength horsepower of helical, double helical with groove or herringbone gear, is given as follows:

Strength horsepower:

$HPst = (Np \times d \times Kv/126000) \times (Fw/K_m) \times (J/P_d) \times Sat \times K_L$

HP_{st}= Strength horsepower

N_p = Pinion speed, RPM

d = pitch diameter of the pinion, inches

K_v= Velocity Factor = $(78/(78+v^{.5}))$^0.5

F_W = Active Face width in inches

K_m= Load Distribution Factor, See Fig.4

J = Geometry Factor, See Fig.5; P_n= Normal Diametral Pitch

Ψ= Helix Angle

P_d= Transverse Diametral Pitch or Diametral Pitch in the plane of rotation, inches

= P_n* Cos Ψ

S_{at}= Allowable Fatigue Stress for material, PSI

K_L= Life Factor for 10^7 cycles of rotation

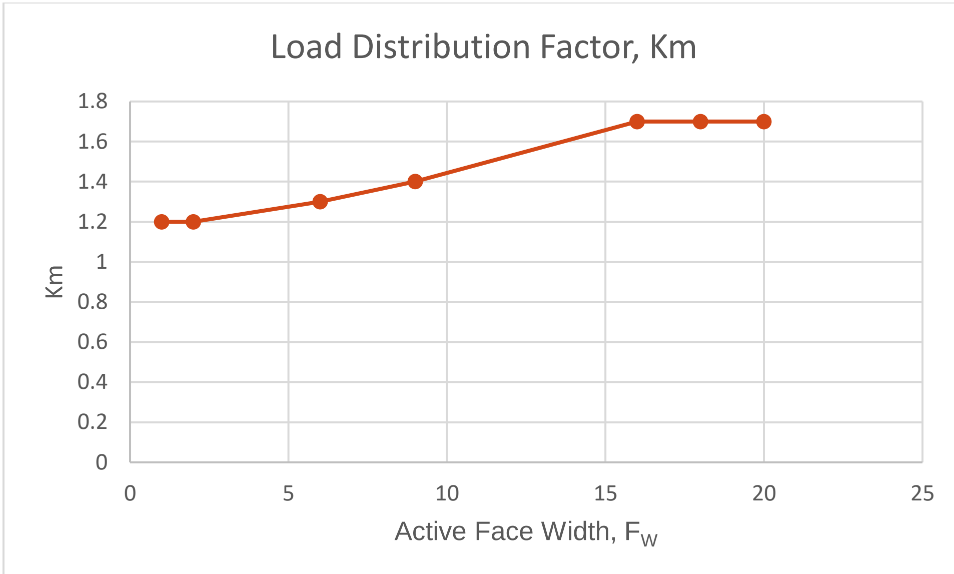


Fig.4: Load Distribution Factor, K_m, Vs. Active Face Width, F_W

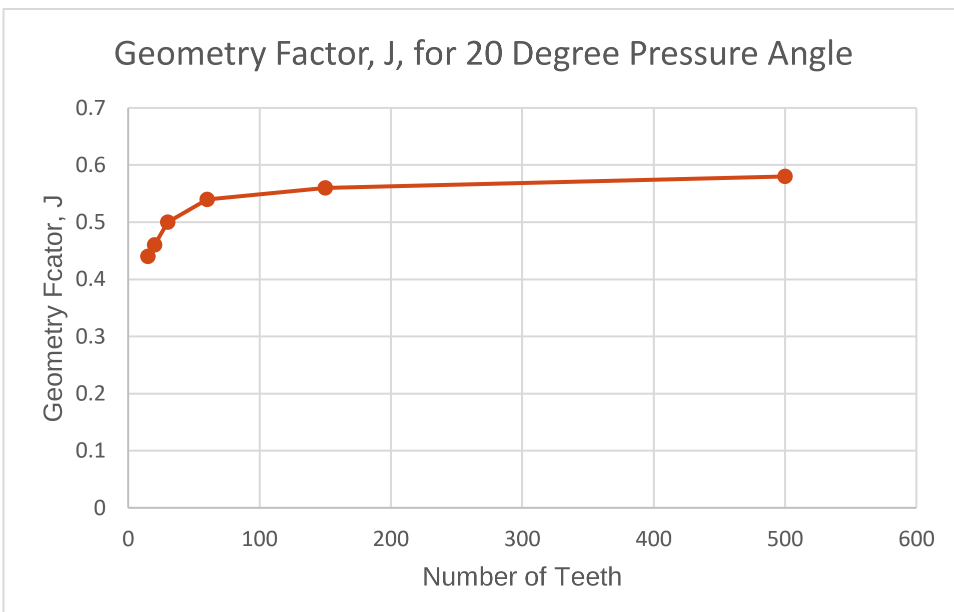


Fig.5. Geometry Fcator, J, For Helix Angle, 10,15, 20 and 30 Degrees

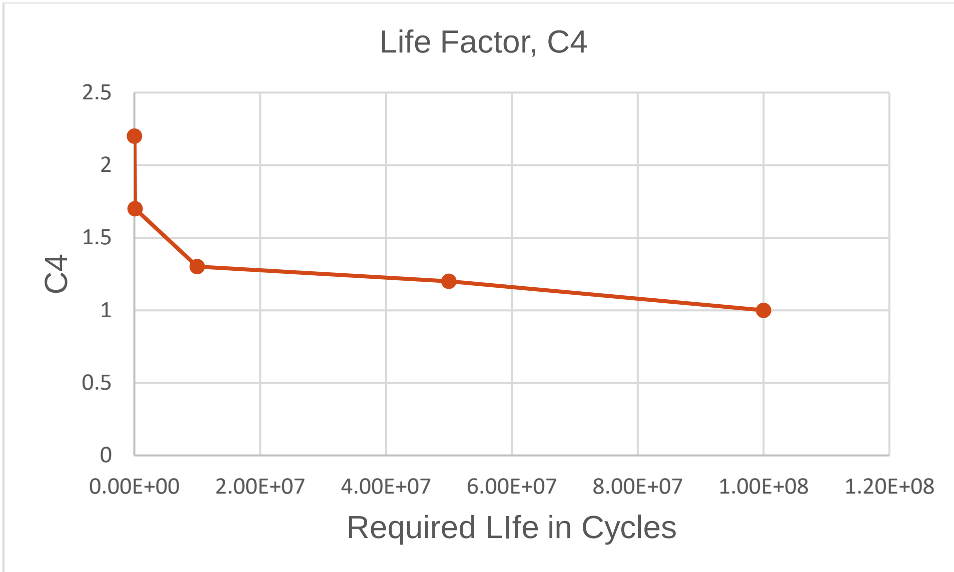


Fig.6. Required Life Factor, C_4

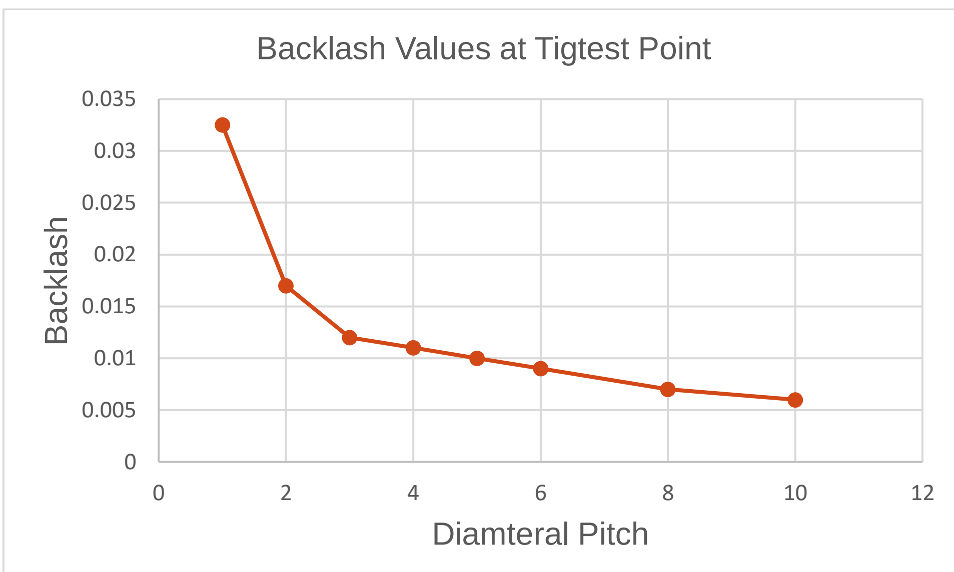
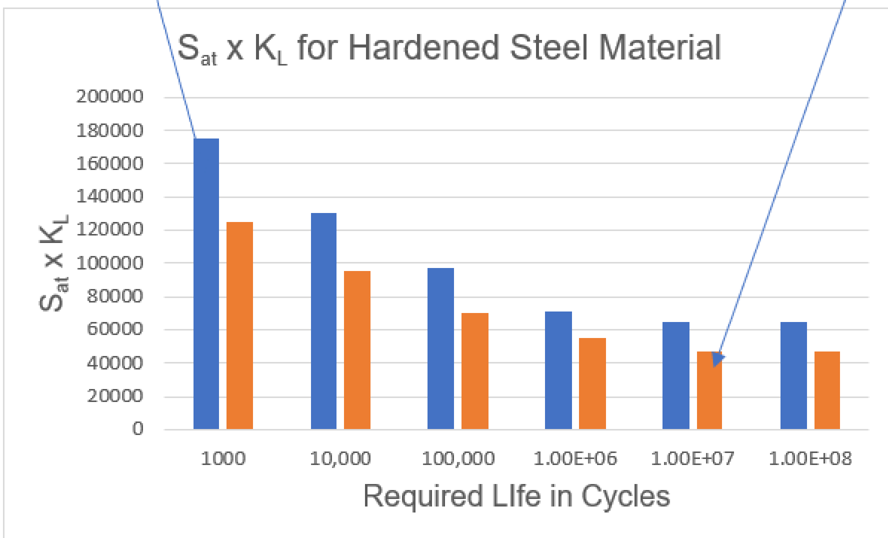


Fig.7. Recommended Backlash Values at assembly at Tightest Point

For Case Carburized, 60 Rockwell Steel

For Induction hardened 52 Rockwell Steel

Fig.8. K_L for



S_{at} x Values

process

Third and Higher Reduction

First Reduction

Second Reduction

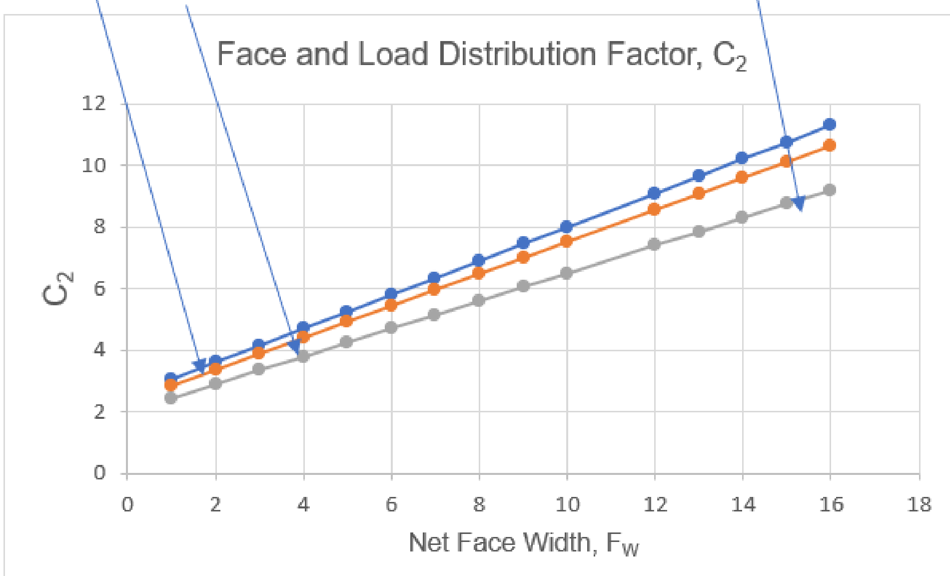


Fig.9. Face and Load Distribution Factor, C_2 Vs. Net Face Width, F_W

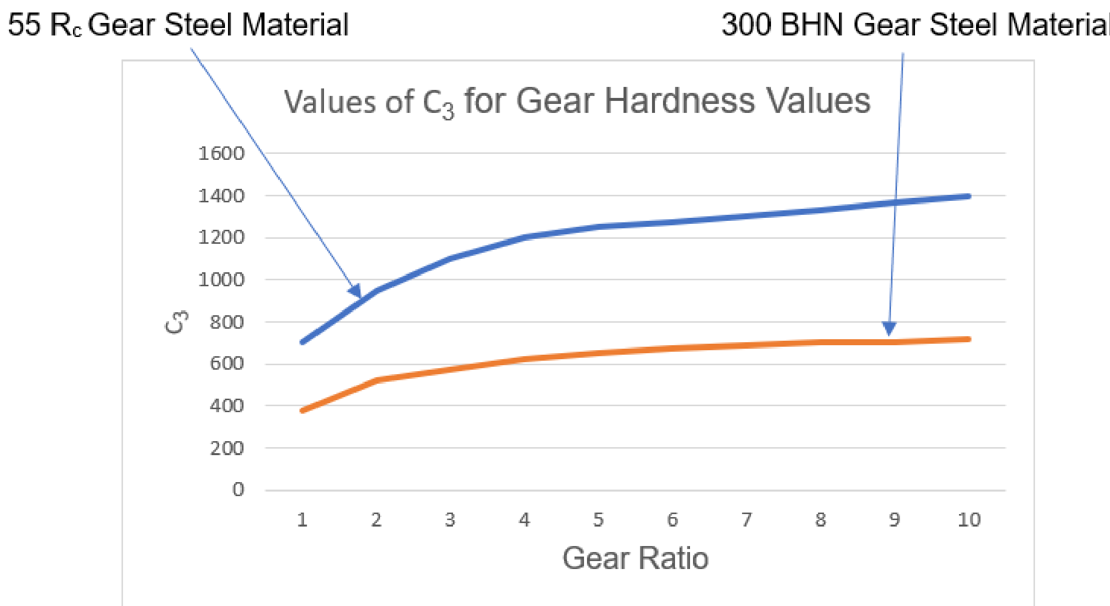


Fig.10. Gear Geometry and Material Factor, C_3 Vs. Gear Ratio, m_G, for External Gear/Pinion Set

(Note: For Class 30 cast Iron, multiply the C_3 values for 300 BHN Steel Values from the above chart by 50%, and for Class 40 Cast Iron, multiply by 75%)

(Note of Caution: These values, tables, multipliers, material data, and details are specific to the quality and manufacturing processes followed by gear companies. They should not be used for gears/pinions manufactured by companies other than Philadelphia Gear Corporation)

(Note: All the above graphs are from the tables recommended by Philadelphia Gear Corporation and have been used successfully by the Author for Gear Box Design for Machine Tool Gear Box)

One of the primary advantages of helical gearing is a higher contact or overlap ratio, i.e., more than one tooth shares the load at any point in time. In other words, tooth proportion, diametral pitch, helix angles, face width, etc., should be selected to increase the overlap ratio. As opposed to spur gear, helical gear carries a higher load due to a higher contact ratio. In particular, face width, diametral pitch and helix angle should be selected such that the highest possible contact ratio is achieved. The overlap ratio must be higher than 1.2 for helical gears. The backlash is required for any gear set and it is measured at the tightest point in the teeth contact. The recommended values are given in Fig.7.

Overlap or Contact Ratio = $(F_W * \tan \Psi * P_d) / 3.1416$

<u>Durability Horsepower Rating:</u>

Durability Horsepower = HP_{du} = (Np x d^2 x Cv/126000) x $C_2 C_3$ x C_4 where

HP_{du}= Durability horsepower

N_p = Pinion speed, RPM

C_V= Velocity Factor = $78/(78+v^{.5})$

V = Pitch Line Velicity, feet/minute or fpm

d = Pitch Diameter of pinion, inches

C_2 = Face and Load Distribution Factor

C_3 = Geometry and Material Factor

C_4 = Life Factor

The service horsepower of the Gear Set = lower of the durability or strength horsepower/service factor. The service factor depends on the run hours, material conditioning or heat treatment, type of loading, gear dimensions, etc.

Example: (Based on author's application of helical Gear design, Machine Tool Application)

<u>Example 1:</u>

<u>Pinion Details:</u>

N_p = Pinion Rpm = 1500

Pinion Pitch Diameter =5.3 inches; Normal Diameter Pitch = 6

Pinion Details: Number of Teeth in Pinion = 30; Pressure angle = 20 degrees; Full Depth, Standard addendum, standard fillet radius, Helix angle = 17 degrees, Left Hand, 3.25 inch Face width, material: SAE 4140 steel with minimum 60 Rockwell hardness.,

Type of Loading: 8-10 Hours per day running, Moderately shock loading. The helical gear set is the first set speed reducer in the gearbox.

<u>Gear Details:</u>

Gear Pitch diameter = 5.3 inches; Normal Diametral Pitch =6, 40 Teeth; 20-degree pressure angle, Full Depth, Standard addendum, standard fillet radius, Helix angle = 17 degrees, Right Hand, 3.25 inch Face width, material: SAE 4140 steel with 60 Rockwell Hardness.

Overlap or Contact Ratio = (F_W * tan Ψ * P_d) / 3.1416 = 3.25 * tan(17)* 5.74

= 3.25*0.306*5.74 = 5.7 (OK)

Strength horsepower:

HPst = (Np x d x Kv/126000) x (Fw/K_m) x (J/P_d) x Sat x K_L

HP_{st}= Strength horsepower

N_p = 1500 RPM

d = pitch diameter of the pinion, inches = 5.3”

V = 3.1416 * 5.3 * 1500/12 = 2081 ft/min

K_v= Velocity Factor = (78/(78+$v^{.5}$))^0.5 =0.79

F_W = Active Face width in inches = 3.25 “

K_m= Load Distribution Factor, See Fig.4 = 1.25

J = Geometry Factor, See Fig.5, = 0.47;

P_n= Normal Diametral Pitch

Ψ= Helix Angle = 17 degrees

P_d= Transverse Diametral Pitch or Diametral Pitch in the plane of rotation, inches = P_n* Cos Ψ = 5.74 inch

S_{at}= Allowable Fatigue Stress for material, PSI

K_L= Life Factor for 10^7 cycles of rotation

S_{at} * K_L = 44,000 from Fig.8

HP_s for Pinion =((1500*5.3*0.79)/126000) * (3.25/1.25) * (0.47/5.74) * 46000

= 0.0498 * 2.6 * 0.082 * 46000 = 488 HP

HP_s for Gear =((1500*5.3*0.79)/126000) * (3.25/1.25) * (0.0.52/5.74) * 43000

= 0.0498 * 2.6 * 0.09 * 43000 = 501 HP

Durability Horsepower Rating:

Durability Horsepower = HP_{du} = (Np x d^2 x Cv/126000) x C_2 C_3 x C_4 where

HP_{du}= Durability horsepower

N_p = Pinion speed, RPM = 1500 rpm

C_V= Velocity Factor = 78/(78+$v^{.5}$) = 78/(78+2081^0.5) = 0.63

V = Pitch Line Velicity, feet/minute or fpm = 3.1416* 5.3 * 1500/12 = 2081 fpm

d = Pitch Diameter of pinion, inches = 5.3"

C_2 = Face and Load Distribution Factor, from Fig.9 =3.0.

C_3 = Geometry and Material Factor, from Fig.10 = 700

C_4 = Life Factor = 1, from Fig.6, 10 x 10e6 cycles

HP_{du} = Durability Horse Power = (Np x d^2 x Cv/126000) x C_2 x C_3 x C_4

= (1500 x 5.3^2 x 0.63/126000) x 3 x 700 x 1

= 442 Horse Power

Service Rating: For service of 8 to 10 hours per day, moderate loading, durability horsepower is the lowest out of strength and durability horsepower. The service factor is 1.3. Hence, durability dictates the gear set life and it is equal to 442/1.3 = 340 HP @ 1500 RPM of the pinion.

12. Gear Tooth Wear and Failure

Material Failures

The author has experienced most of the tooth failures or excessive wear due to selecting the wrong or weaker gear material that cannot sustain the duty cycle loads. When the material endurance strength is not enough to sustain the fatigue loadings, the tooth wear becomes excessive. When shock load or excessive static load is present, teeth fail abruptly before the expected design life. In some cases, permanent tooth deformation also happens in many cases. Friction goes up, and excessive heat is created. Such abnormal heat also deforms the tooth prematurely. Tooth flank wear or the failure at the tooth tip also occurs. Shock loading also creates a failure termed "Peening." Fatigue failures create another type of tooth failure below the pitch line, called "Spalling." Spalling happens due to fatigue failure of the teeth. Fatigue failure of the material is due to poor tooth hardness, and an increase in tooth hardness will minimize such failures to a great extent. Fatigue cracks appear on the tooth flank, and cracks develop along the tooth surface. Material selection is very important for gear life.

Lubrication Failure

Failure also happens due to poor heat management and poor lubrication for the gear set. Due the poor lubrication, the gear develops scars on the surface and galling of the tooth. The failure starts from the tip of the tooth. The lubrication causes burnishing of the teeth and causes a mirror finish of the tooth. Discoloration could be due to poor lubrication or poor heat dissipation. Failure also occurs due to lubricant contaminants that consist of gear materials and cast-iron inclusions such as sand, core material, etc.

Tooth Surface Finish

The tooth contacts another tooth during meshing along the pitch line. The impact due to contact happens at this point along the pitch line. The surface finish of the teeth must be smooth enough so that the load is shared across the face uniformly. In case the surface has high spots, these spots take higher impact loads and have a much higher chance of failure. Hence the surface finish must be as recommended for the application by the gear manufacturers. For most of the machine tool applications, teeth are hobbed and grounded.

Effects of Contamination

I have experienced tooth failure due to contaminants in the lubricants coming from casting or internal and external sources. Any particle or grinding dust can affect the tooth's life. Lubricant must be filtered and changed frequently to enhance tooth life. The particles get embedded in the gear wheel teeth more often and affect the gear life. These particles become like grinding points and create scratch marks on the tooth flanks. Normally gear wheel gets affected much sooner than the worm since the worm is hardened.

Load Impact

If the mechanical load is repetitive with high-impact conditions, a tooth might fail abruptly. Once a tooth or two fails, the failure progresses very quickly, and the system fails. The tooth design has to be properly done to sustain a high-impact load. Fortunately, loading in turning, milling, or grinding machines is somewhat uniform, and worm gears are very suitable. Indexing applications do see some impact loading during starting and stopping conditions due to high accelerations and decelerations, but I had no problem with tooth failures under such conditions. Impact loading also causes tooth deformation, and meshing becomes irregular, creating a clunk sound. Tooth backlash increases, and teeth engagement becomes irregular. The friction also increases creating more heat due to excessive rubbing conditions or sliding conditions.

13. Single Helical Gear Force Analysis

The following discussion is used to determine the forces developed during gear meshing. The determination of these forces is required to design gears, bearings, and gear shafts. There are primarily three types of steady or variable forces generated during tooth meshing: Radial to the gear tooth, Tangential to the pitch line, and Axial force along the axis of the shaft holding the gears. For spur gear axial force is not generated. The gear shaft and supporting bearings have to be designed to resist all these gear forces within the design life. In typical cases, the shaft or bearing fails much earlier than the gear itself. Tangential force is created due to torque transmission, and radial force, due to involute tooth form, is directly transmitted to the shaft. The design of end supporting bearings will also depend on these types and amount of these forces. Sometimes, the axial force is also created due to misalignment of the shafts. Let us examine the type and nature of the forces for different helical gearing arrangements. (See Fig.11)

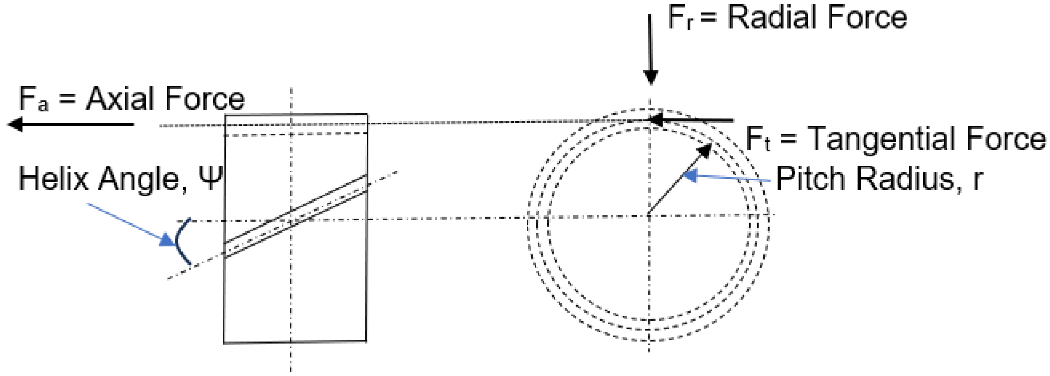


Fig.11. Tooth Forces for Single Tooth Helical Gear

α = Normal Pressure Angle, Degrees = 20 degrees
r = Pitch Radius= 4 inch
F_t = Tangential Force = Transmitted Torque/Pitch Radius = T/r
F_r = Radial Force or Separating force = F_t * Tan α / Cos Ψ
F_a = Axial Force = F_t Tan Ψ

Say, Transmitted Horsepower = 100 HP and Pinion RPM, N = 1500
HP = N * T/63000 = 100; so, T, Torque Transmitted = 100* 63000/1500
T = 4200 in-lbs

F_t = Tangential Force = Transmitted Torque/Pitch Radius = 4200/4 = 1050 lbs
F_r = Radial Force or Separating force = F_t * tan α / Cos Ψ = 1050*tan20/cos17
= 1050*.36/.95 = 400 lbs
F_a = Axial Force = F_t tan Ψ = 1050 * 0.305 = 321 lbs

14. Steps for Gear Design and Analysis

Next, let me share with you the design process for shaft and bearing design that I have used over my professional years and have been very successful in doing that for a continued period. The author has taken one such example from my old notes and reproduced it here. This is also to remind the readers that this process is not the only recommended practice to reach the end and this is what I have done for designing a speed reducer for one of the applications that came on my way. Basically, for any gearbox design, three typical analyses have been performed:

- Gear Capacity Analysis to satisfy requirements
- Gear Support Shaft Stress and Life Analysis
- Gear Shaft Support Bearing Analysis

In addition to the above analysis, the author has also calculated housing stress analysis due to heat and impact loading using Finite Element Analysis. There is no scope for reproducing that type of analysis in this book due to shortage of space and other limitations. First of all, we have to determine and understand the requirements for such a speed reducer. During my tenure in the industry, I always used to get help from industry experts who are involved with design and analysis daily. I always followed the life calculations recommended by Prof. Shigley. I always consulted engineers from the following companies:

- Philadelphia Gear Corporation
- RHP Bearing Corporation

In general, successful gear design depends on various factors such as Load variations, peak load, reliability desired, life required, risk factors, allowable face width, diametral pitch, type of tooth, gear ratio desired, mesh material, and heat treatment process applied to name a few.

In every possible case, surface strength and durability strengths need to be calculated using safety factors desired for the applications. In addition to strength and durability analysis, bending of the tooth, deflection of the shaft, etc. need to be calculated also. Tooth bending depends on the material, hardness values, factor of safety desired, and face width. To resist pinion and gear wear, surface hardness is very important. Tooth surface finish is equally important. For flame-hardened gears, surface hardness, depth of hardness, etc., are very important.

15. Shaft and Bearing Analysis

For a successful Gear design, all the components of the assemble must be designed and analyzed properly. It has been seen very often the components fail before the gear or pinion fails. In addition to the gear failure, improper system design might cause excessive noise, premature gear failure, supporting bearing and shaft failures, shaft deflection and misalignment, gear pitting, etc. The shaft deflection also depends on the proper selection of the support bearings. The deflection of the shaft under the load depends on the shaft and bearing stiffness in addition to the tooth load and torque transmitted. In this section, the shaft and bearing design will be explored. (See Fig. 12 also)

Shaft Stress Analysis: The gearing arrangement for the pinion is assumed as given below:

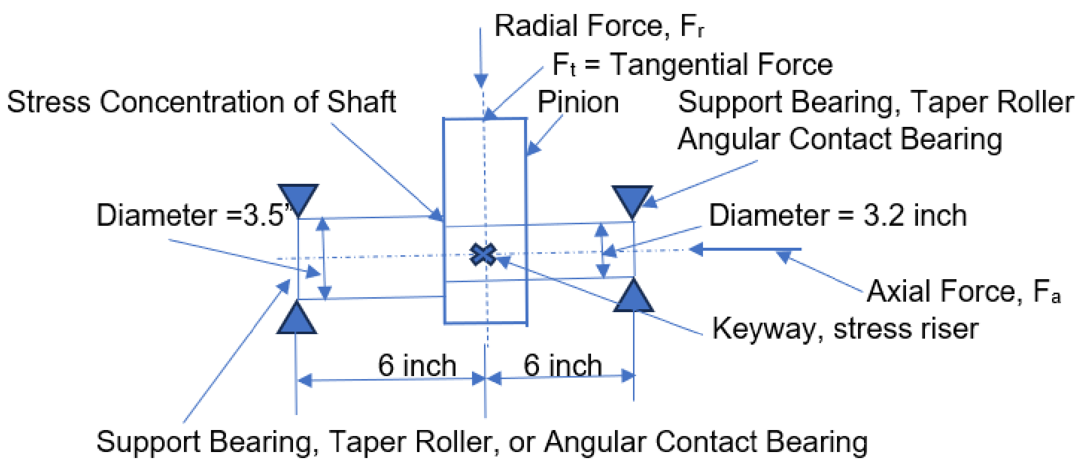


Fig.12: Shaft and Pinion Arrangement for Force Analysis

F_{res} = Resultant Transverse force on shaft = $(F_t\text{^}2 + F_r\text{^}2)\text{^}0.5$

=(1050^2 +400^2)^.5 = 1124 lbs

Deflection Analysis of Shaft: Due to the radial force created during meshing, the shaft will deflect. It is important to design the shaft with minimum deflection to avoid misalignment and whipping of the shaft during running. The shaft is considered to be a simply supported beam with load in the middle, as shown in Fig.12. To have a higher factor of safety, we will assume the shaft has a 3.5-inch diameter throughout the span. The shear force, bending moment diagram, and elastic deflection curve for the beam are shown in Fig.13 below.

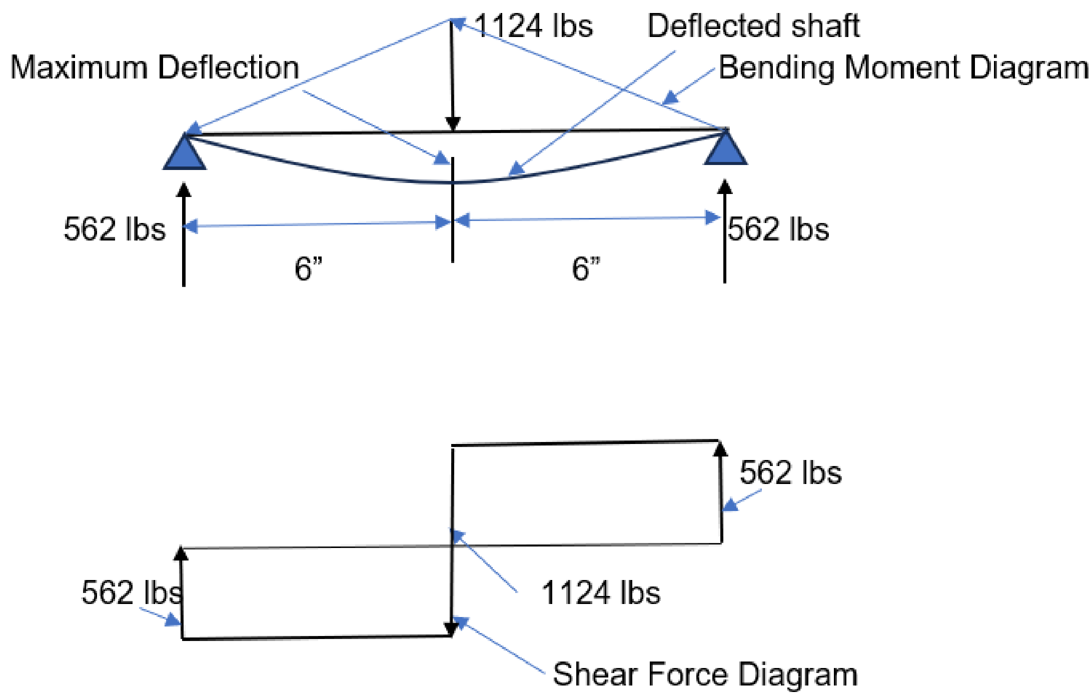


Fig.13: Shear Force, bending Moment, and deflection for the beam

For a supported uniform beam, the maximum deflection, δ_{max}, at the middle of the shaft, from any Machine Design Handbook, is as follows:

δ_{max} = F_R * L^3/(48 EI) where

I = Moment of inertia at middle = (3.14*d^4)/64
= (3.14 * 3.5^4)/64 = 7.36 in^4
E = 30E6 psi for steel material
L = 12 inch = span of the shaft between bearings

δ_{max} = F_{res} * L^3/(48 EI) = (1124 * 12^3)/(30e6*7.36)
= 0.008 inch

Next, we have to calculate the bending and shear stress of the beam. The shear stress, τ_{xy}, is due to the torque applied to the beam during running and bending stress. σ_b is due to the bending of the beam under concentrated loading.

Principal Stress Analysis of the Beam: After calculating the bending and shear stress, principal stresses can be calculated also as explained below:

Bending Stress, σ_b = (M/I) * d/2 = (562*6/7.36)*(1.75) = 800 psi
Shear Stress, τ_{xy} = (T/J)* d/2 = (4200/(2*7.36)) * 1.75 = 500 psi

Principal Stresses, σ_1, σ_2 = σ_b/2 +/- ((σ_b/2)^2 + τ_{xy}^2)^0.5 i.e.
σ_1 = 800/2 + ((400^2+250^2)^0.5 = 400 + 472 = 872 psi, tensile force
σ_2 = 400- 472 = -72 psi, compressive stress

16. Crossed Helical Gears

For helical and spur gear arrangements, shafts are parallel to each other. Crossed helical gears is a special helical gear where the shafts are not parallel to each other. Crossed helical gears transmit power when the shafts cannot be parallel to each other. For such arrangements, tooth sliding during transmission occurs heavily and heat is generated due to such sliding of teeth. Contact between teeth is very limited. Lubrication is very critical for such gear arrangements. When the shaft has to rotate clockwise and counterclockwise, the selection of helix angle is very critical. This type of helical gear can be used to transmit a limited amount of power when the shafts are not parallel or non-intersecting but can be assembled without many problems. In addition, this type of gear is used for a single first stage of reduction. For larger ratios, worm gears are much more suitable than the crossed helical gear. For heavy power transmission, bevel gears or worm gears should be used. (See Fig.15 for teeth details)

Because of severe sliding action during power transmission for these gears, the selection of material and lubrication process becomes very critical for successful operation. Such selection is required to make sure that gears do not seize due to overheating due to sliding action and heavy friction between teeth. Steel materials are suitable for such types of gear. (See Fig.14 also)

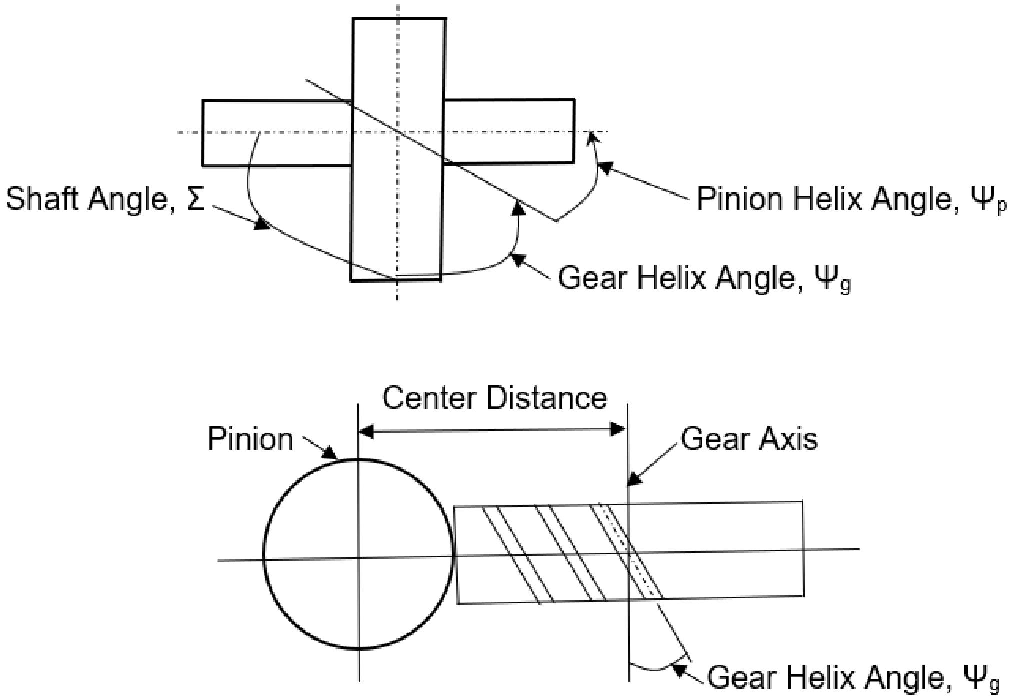
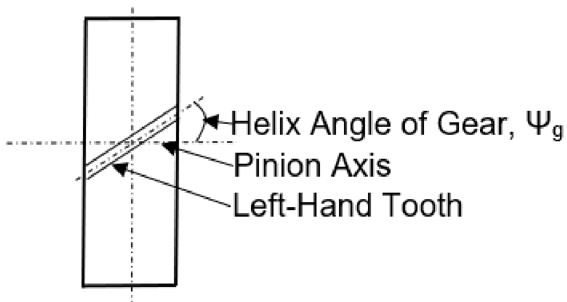


Fig.14. Crossed Helical Gear Arrangement Details



Ψ = Helix Angle; P_n = Diametral Normal Pitch = Diametral Pitch of Cutter

P_d = Diametral Normal Linear; N = Number of Teeth; a = addendum

B = dedendum; h_t = whole depth of tooth; D = Pitch Diameter

D_o = Outside diameter; $\o_n$ = Normal Pressure Angle

$\Sigma = \Psi_p + \Psi_g$ = Shaft Angle

$D_p = N_p/(P_n * \cos\Psi_p)$

$D_g = N_g/(P_n * \cos\Psi_g)$

Center Distance, $C = (D_p + D_g)/2$

M_g = Gear Ratio = N_g/N_p

Fig.15. Crossed Helical Gear Arrangement Details (Source: Philadelphia Gear Corporation Gear Handbook)

17. Causes and Mitigation of Gear Noise

At the outset, the author must admit that he is not a gear noise expert by any stretch of your imagination, but he has witnessed and observed various types of noises in a gearbox or indexing unit. Gear noise is the result of impact between teeth while engaging and disengaging from the contact. One tooth creates impact while meshing with the other. Normally, constant mesh gear arrangements such as worm and worm wheel are very quiet since engagement and disengagement of teeth are very gradual by design since the contact ratio is way more than 1. On the other hand, spur gears and straight-tooth bevel gears are noisy. Spiral bevel gears or helical gears tend to emit less noise. The high-pitched gearbox noise, called whining noise, could be detrimental to the health of the operator. The primary cause of noise in a gear arrangement or gearbox could be numerous, as outlined below.

- Poor Gear Quality: It is one of the primary causes of noisy gears. The gear tooth tolerances and tooth forms must be very accurate and manufactured as per AGMA specifications. Geometrical differences between teeth create the impact condition. Non-confirming tooth profiles create noise in most of the cases. Gear sets must be changed as a pair. Surface finish, gear materials, manufacturing methods, hardness process, etc., are the parameters of good gear quality. To have good gear quality, several factors, such as profile tolerances, tooth pitch tolerance, pitch-line runout, Lateral runout, etc., must be controlled very precisely and as per AGMA standards.

- Excessive Shaft Vibration: If shaft stiffness is not enough for the speed, resonance condition exists, and that creates noise or whirling sound. Shaft resonance must not be allowed for high-speed gearboxes. Resonance conditions could also exist when the input speed conditions create resonance with the natural vibration of the gear shafts. Both natural and forced vibration conditions must be checked for the design.

- Excessive Tooth Deflection: If the tooth deflection is more than what is allowed for the load transmission, the tooth creates noise conditions. Teeth are not able to transmit load at the desired speed. Higher tooth thickness must be selected in such cases.

- Shaft Misalignment: For parallel shafts, the center distance between shafts must be tolerated properly so that shafts are parallel within acceptable limits. Center distance tolerances must be checked and installed properly. Center distance must be selected properly so that excessive backlash or tooth interferences are avoided.

- It might also create premature failure of gear teeth due to excessive backlash or tooth interference conditions. Shaft runout must also be checked while assembling the shafts. Center distance tolerance is a function of several factors, such as backlash requirements, tooth pitch, gear class selected, etc.

- Excessive Backlash: When gears wear out, the backlash is created, and gears create a "ping-ping" sound during engagement. Gears must have some backlash to work properly, but excessive backlash is detrimental to gear functioning and gear life. AGMA recommends a backlash determined by the equation: Backlash, b = center distance x 2 x tan (pressure angle). Hence, center distance is directly dependent on the backlash requirements between gear teeth.

- Center Distance: As mentioned before installed center distance between shafts is very critical for proper gear functioning. Several factors, such as shaft deflections, tooth engagements, temperature, distortions, etc., depend heavily on how close the center distance is controlled. For a very large gearbox, maintaining near-perfect center distance is a challenge. Sometimes, gearbox wall deflection and distortion also create center distance variations.

18. Summary

- Gear specifications must include details such as Circular Pitch, Diametral Pitch, Module for metric gears, Addendum, Dedendum, Tooth height, Outside diameter, pitch circle diameter, whole depth, backlash, etc. These specifications can be taken from AGMA Standards also. The details of specifications must follow the standards for the specific gear and pinion.

- Primary causes of gear noise are Center Distance variations, Shaft Resonance, Poor Quality Gears, Poor Surface finish, Tooth Interference, excessive backlash, shaft misalignment, unbalance of gears, excessive shaft or tooth deflection, etc.

- Gear Box Analysis should consist of Gear life and stress analysis, Shaft Stress and life analysis, housing thermal distortion analysis, heat dissipation analysis, Support bearing analysis, Material Strength analysis, shaft resonance analysis, etc.

- Gear Force analysis should consist of shaft deflection analysis due to loading, tooth deflection analysis due to power transmission, shaft force analysis, support bearing analysis, etc.

- Gear Tooth failure could be attributed to several factors, such as poor material selection, lubrication failure, excessive heat, shaft misalignment, poor surface finish, improper tooth hardness, tooth interference, excessive tooth backlash, excessive impact loading, etc.

- The service rating of any gear set depends on maximum sliding velocity or surface speed, tooth form and hardness, tooth surface finish, lubrication method, shaft deflection, and shaft resonance during load transfers. Most of this data could be obtained from the gear manufacturers.

- Gearbox design should consist of several sequential steps, such as the determination of strength rating, durability, horsepower rating, etc., for both the gear and pinion. Proper service and safety factors must be applied to the analysis to get a realistic value for the life of the gear and its service rating. The lower of the two-horsepower ratings (Durability and strength) is the service rating of the set, which must be higher than the required horsepower to be transmitted.

- Almost all the gears in machine tool applications are heat-treated to increase load-carrying capacity. Gear hardening is obtained either by the surface hardening process or through the hardening process. Thru hardening of steel is done using oil quenching to a specified hardness. Hardening can also be done using carburizing or nitriding process. The surface hardening is done to enhance the wear rate of the gears. Flame hardening is also used to harden the gears depending on the gear materials.

19. Suggested Readings

1. Blackburn, J.D., Acherkan, N., Machine tool Design, Volume 1, MIR Publisher, Moscow 1968., NY
2. Philadelphia Application Engineered Gearing: Philadelphia Gear Corporation, Pennsylvania 18406
3. Shigley, J.E., Mechanical Engineering Design, Third Edition, McGraw Hill Book Corporation, ISBN 0-07-056881-2
4. RHP Super Precision Bearing Catalogue, C761, England, RHP Incorporated, Elyria, Ohio, USA
5. Oberg, E., Jones, F.D., Horton, H.L. and Ryffel, H.H., Machinery's handbook- 27th Edition, 2004, Industrial Press Incorporated, NY
6. Chironis, N.P., Senior Editor, Gear Design and Application, McGraw-Hill Book Company, NY
7. Dudley, D.W. Editor, First Edition, Advance Gear Engineering, General Electric Company, Lynn, Massachusetts, McGraw-Hill Book Company, NY
8. AGMA 390.02, Gear Handbook, published by American Gear Manufacturing Association, September 1964
9. AGMA 390.01, Gear Handbook, published by American Gear Manufacturing Association, September 1964