Soil Mechanics Series - Stress and Strain

An Online Continuing Education Course for Engineers

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Soil Mechanics Series: Stress and Strain

Soil mass is generally a three phase system that consists of solid particles, liquid and gas. The liquid and gas phases occupy the voids between the solid particles as shown in Figure 1a. For practical purposes, the liquid may be considered to be water (although in some cases the water may contain some dissolved salts or pollutants) and the gas as air. Soil behavior is controlled by the interaction of these three phases. Due to the three phase composition of soils, complex states of stresses and strains may exist in a soil mass. Proper quantification of these states of stress, and their corresponding strains, is a key factor in the design and construction of transportation facilities.

The first step in quantification of the stresses and strains in soils is to characterize the distribution of the three phases of the soil mass and determine their inter-relationships. The inter-relationships of the weights and volumes of the different phases are important since they not only help define the physical make-up of a soil but also help determine the in-situ geostatic stresses, i.e., the states of stress in the soil mass due only to the soil's self-weight. The volumes and weights of the different phases of matter in a soil mass shown in Figure 1a can be represented by the block diagram shown in Figure 1b. Such a diagram is also known as a phase diagram. A block of unit cross sectional area is considered. The symbols for the volumes and weights of the different phases are shown on the left and right sides of the block, respectively. The symbols for the volumes and weights of the three phases are defined as follows:

V_a, W_a : volume, weight of air phase. For practical purposes, $W_a = 0$.

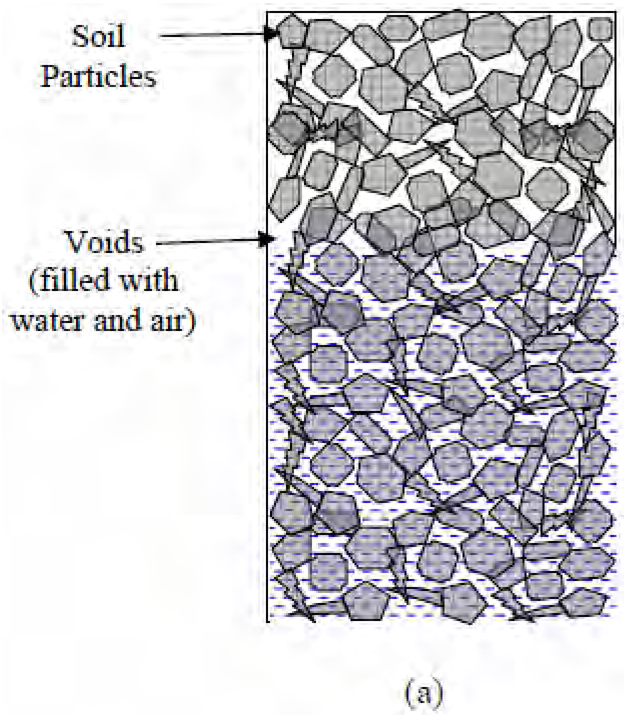
V_w, W_w: volume, weight of water phase.

V_v, W_v : volume, weight of total voids. For practical purposes, $W_v = W_w$ as $W_a = 0$.

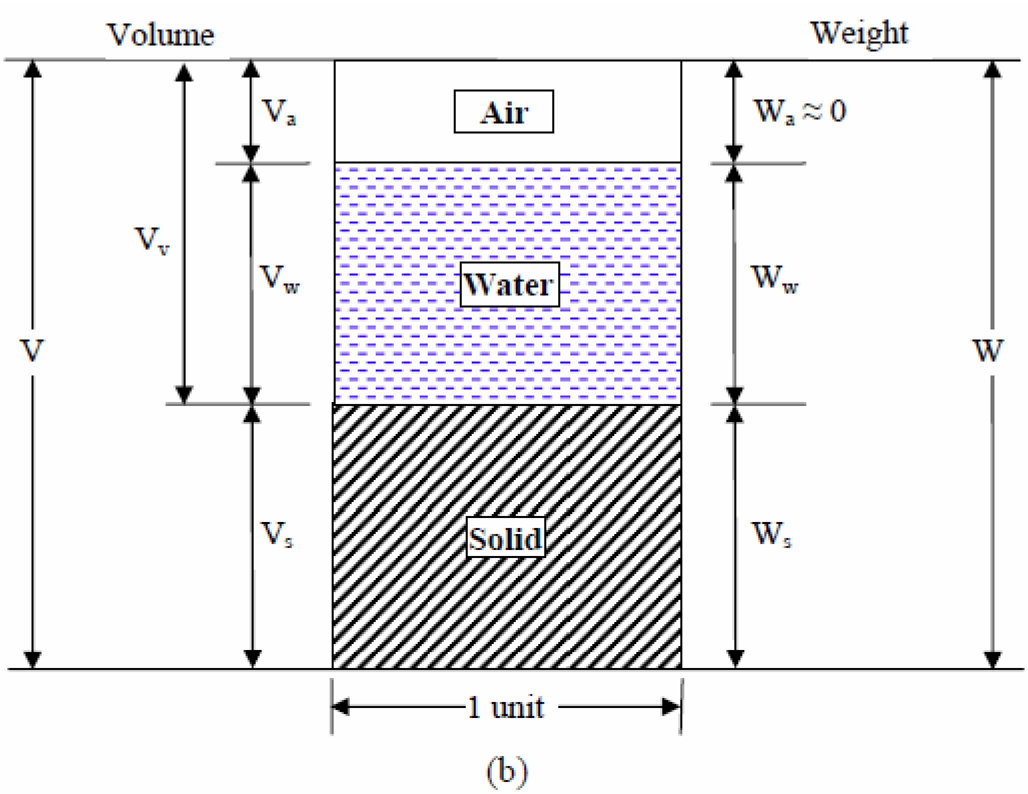
V_s, W_s : volume, weight of solid phase.

V, W : volume, weight of the total soil mass .

Although $W_a = 0$ so that $W_v = W_w$, V_a is generally > 0 and must always be taken into account. Since the relationship between V_a and V_w usually changes with groundwater conditions as well as under imposed loads, it is convenient to designate all the volume not occupied by the solid phase as void space, V_v. Thus, $V_v = V_a + V_w$. Use of the terms illustrated in Figure 1b, allows a number of basic phase relationships to be defined and/or derived as discussed next.



(a)



(b)

Figure 1. A unit of soil mass and its idealization.

BASIC WEIGHT-VOLUME RELATIONSHIPS

Various volume change phenomena encountered in geotechnical engineering, e.g., compression, consolidation, collapse, compaction, expansion, etc. can be described by expressing the various volumes illustrated in Figure 1b as a function of each other. Similarly, the in-situ stress in a soil mass is a function of depth and the weights of the different soil elements within that depth. This in-situ stress, also known as overburden stress, can be computed by expressing the various weights illustrated in Figure 1b as a function of each other. This section describes the basic inter-relationships among the various quantities shown in Figure 1b.

Volume Ratios

A parameter used to express of the volume of the voids in a given soil mass can be obtained from the ratio of the volume of voids, V_v, to the total volume, V. This ratio is referred to as **porosity, n,** and is expressed as a percentage as follows:

$$n = \frac{V_v}{V} x100 \qquad (1)$$

Obviously, the porosity can never be greater than 100%. As a soil mass is compressed, the volume of voids, V_v, and the total volume, V, decrease. Thus, the value of the porosity changes. Since both the numerator and denominator in Equation 1 change at the same time, it is difficult to quantify soil compression, e.g., settlement or consolidation, as a function of porosity. Therefore, in soil mechanics the volume of voids, V_v, is expressed in relation to a quantity, such as the volume of solids, V_s, that remains unchanging during consolidation or compression. This is done by the introduction of a quantity known as **void ratio, e,** which is expressed in decimal form as follows:

$$e = \frac{V_v}{V_s} \qquad (2)$$

Unlike the porosity, the void ratio can have values greater than 1. That would mean that the soil has more void volume than solids volume, which would suggest that the soil is "loose" or "soft." Therefore, in general, the smaller the value of the void ratio, the denser the soil. As a practicality, for a given type of coarse-grained soil, such as sand, there is a minimum and maximum void ratio. These values can be used to evaluate the **relative density, D_r (%),** of that soil at any intermediate void ratio as follows:

$$D_r = \frac{(e_{max} - e)}{(e_{max} - e_{min})} x100 \qquad (2a)$$

At $e = e_{max}$ the soil is as loose as it can get and the relative density equals zero. At $e = e_{min}$ the soil is as dense as it can get and the relative density equals 100%. Relative density and void ratio are particularly useful index properties since they are general indicators of the relative strength and compressibility of the soil sample, i.e., high relative densities and low void ratios generally indicate strong or incompressible soils; low relative densities and high void ratios may indicate weak or compressible soils.

While the expressions for porosity and void ratio indicate the relative volume of voids, they do not indicate how much of the void space, V_v, is occupied by air or water. In the case of a saturated soil, all the voids (i.e., soil pore spaces) are filled with water, $V_v = V_w$. While this condition is true for many soils below the ground water table or below standing bodies of water such as rivers, lakes, or oceans, and for some fine-grained soils above the ground water table due to capillary action, the condition of most soils above the ground water table is better represented by consideration of all three phases where voids are occupied by both air and water. To express the amount of void space occupied by water as a percentage of the total volume of voids, the term **degree of saturation, S,** is used as follows:

$$S = \frac{V_w}{V_v} x100 \qquad (3)$$

Obviously, the degree of saturation can never be greater than 100%. When S = 100%, all the void space is filled with water and the soil is considered to be **saturated**. When S = 0%, there is no water in the voids and the soil is considered to be **dry**.

Weight Ratios

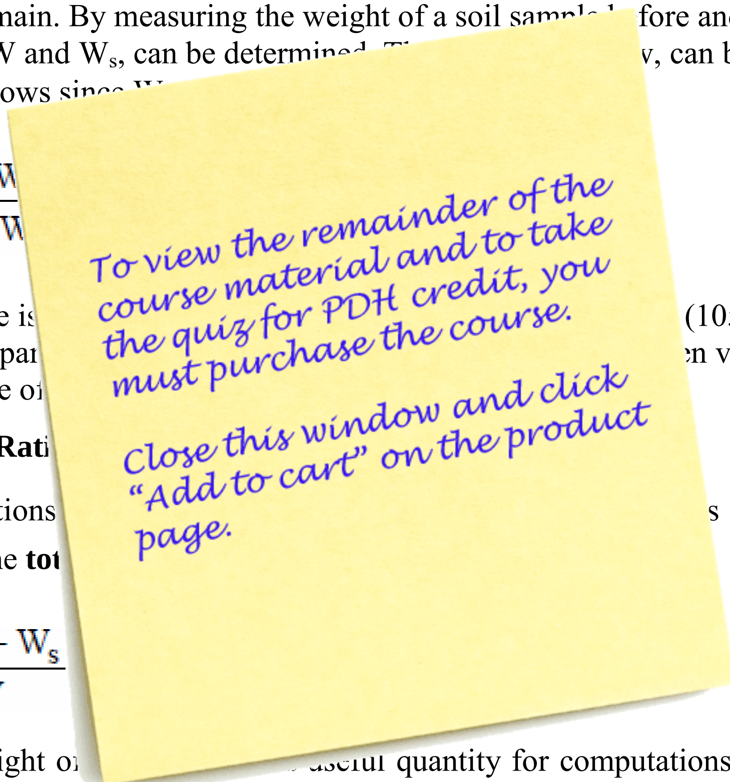
While the expressions of the distribution of voids in terms of volumes are convenient for theoretical expressions, it is difficult to measure these volumes accurately on a routine basis. Therefore, in soil mechanics it is convenient to express the void space in gravimetric, i.e., weight, terms. Since, for practical purposes, the weight of air, W_a, is zero, a measure of the void space in a soil mass occupied by water can be obtained through an index property known as the ***gravimetric*** **water or moisture content, w,** expressed as a percentage as follows:

$$w = \frac{W - W_s}{W_s} = \frac{W_w}{W_s} x100 \qquad (4)$$

The word "gravimetric" denotes the use of weight as the basis of the ratio to compute water content as opposed to volume, which is often used in hydrology and the environmental sciences to express water content. Since water content is understood to be a weight ratio in geotechnical engineering practice, the word "gravimetric" is generally omitted. Obviously, the water content can be greater than 100%. This occurs when the weight of the water in the soil mass is greater than the weight of the solids. In such cases the void ratio of the soil is generally greater than 1 since there must be enough void volume available for the water so that its weight is greater than the weight of the solids. However, even if the water content is greater than 100%, the degree of saturation may not be 100% because the water content is a weight ratio while saturation is a volume ratio.

For a given amount of soil, the total weight of soil, W, is equal to $W_s + W_w$, since the weight of air, W_a, is practically zero. The water content, w, can be easily measured by oven-drying a given quantity of soil to a high enough temperature so that the amount of water evaporates and only the solids remain. By measuring the weight of a soil sampl… fore and after it ahs been oven dried, both W and W_s, can be determin… v, can be determined as follows sin…

$$w = \frac{W - W_s}{W_s} = \frac{W \ldots}{W \ldots} \qquad (4a)$$



Most soil moisture i… (105 and 110°C). Therefore, to compar… n various soils and projects, this range o…

Weight-Volume Rat…

The simplest relations… s (refer to Figure 1b) is known as the **tot…**

$$\gamma_t = \frac{W}{V} = \frac{W_w + W_s}{V} \qquad (5)$$

The total unit weight o… useful quantity for computations of vertical in-situ stresses. For a constant volume of soil, the total unit weight can vary since it does not account for the distribution of the three phases in the soil mass. Therefore the value of the total unit weight for a given soil can vary from its maximum value when all of the voids are filled with water (S=100%) to its minimum value when there is no water in the voids (S=0%). The former value is called the **saturated unit weight,** γ_{sat}; the latter value is referred to as the **dry unit weight**, γ_d. In terms of the basic quantities shown in Figure 1b and with reference to Equation 5, when $W_w = 0$ the **dry unit weight,** γ_d, can be expressed as follows:

$$\gamma_d = \frac{W_s}{V} \qquad (6)$$