Symmetrical Components

An Online Continuing Education Course for Engineers

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Symmetrical Components

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Introduction

The analysis of a steady-state three-phase electrical system is relatively simple to perform by using a per-phase equivalent circuit. This method is only valid though when the voltages and currents are balanced (i.e. equal magnitudes and displaced 120 degrees apart) and when each phase has the same impedance. The analysis becomes much more difficult when the phase voltages and currents are unbalanced such as occurs during unbalanced faults. Examples of unbalanced faults include single-line-to-ground, double-line-to-ground, and line-to-line faults.

In 1918, Charles Fortescue presented the now classic paper "Method of symmetrical component coordination applied to the solution of polyphase networks" to the American Institute of Electrical Engineers (now known as IEEE). While his approach is useful for any number of phase conductors, this discussion is limited to three-phase networks.

The basic premise of *symmetrical components* is that an unbalanced network of three related vectors can be resolved into three sets of vectors. Two of the sets have equal magnitude and are displaced 120 degrees apart while the third set has equal magnitude, but zero phase displacement. The three sets are known as the positive, negative, and zero sequence components of the electrical system.

To study the use of symmetrical components we will first review the math that is used in solving symmetrical component equations and the application of per-unit calculations to electric power systems. Then we will study system components in detail including component schematics and network connections. Finally, we will use an example to bring it all together. But first, a math review.

I. Complex Math

Using symmetrical components to analyze unbalanced electric systems is rather straightforward, but it does require a good understanding of complex vector notation and manipulation. Before delving into symmetrical components we need to review polar/rectangular coordinates, the "α" operator, and matrix multiplication.

Polar and Rectangular Coordinates

A vector written in polar coordinates can be resolved into its rectangular coordinates using simple trigonometric equations. Given a polar coordinate in the form of "$r\angle\theta$" the equivalent rectangular coordinates are found as:

$$x = r * \cos(\theta)$$

$$y = r * \sin(\theta)$$

In complex notation the resulting answer is in the form of x+jy.

Likewise, rectangular coordinates can be converted into polar coordinates using the following equations:

$$r = (x^2 + y^2)^{0.5}$$

$$\theta = \tan^{-1} (y/x)$$

Where,
x = rectangular x-axis.
y = rectangular y-axis.
r = Polar resultant, vector.
θ = Polar angle, degrees.

Note that the angles are represented in degrees and not radians in these examples. As an example of the calculations consider a polar coordinate of $5\angle 53.13$. The rectangular coordinates are:

$x = 5 * \cos(53.13)$

$x = 3.$

$y = 5 * \sin(53.13)$

$y = 4.$

Expressing the resulting answer as a complex number yields 3+j4.

Next consider a complex number of 1+j1.732. What is the polar equivalent?

$r = (1^2 + 1.732^2)^{0.5}$

r = 2.

$\theta = \tan^{-1}(1.732/1)$

θ = 60.

Therefore, the polar equivalent is 2∠60.

When converting from rectangular coordinates to polar coordinates, care must be taken in deciding what angle is being presented. Consider the figure on the left, which shows an example of rectangular coordinates in each quadrant. Point "B" in Figure 1 is -1+j1. Using the formulas just presented, the polar equivalent is 1.414∠-45, but the angle is referenced from the negative x-axis. When referenced from the positive x-axis the angle is 135 degrees. Most handheld calculators will report 135 degrees for this scenario. This location can also be referenced as -225 degrees from the x-axis. The following table shows the results for calculations in each of the four quadrants shown in Figure 1 along with a typical handheld calculator results and finally the results for angles measured from the positive x-axis in the positive direction.

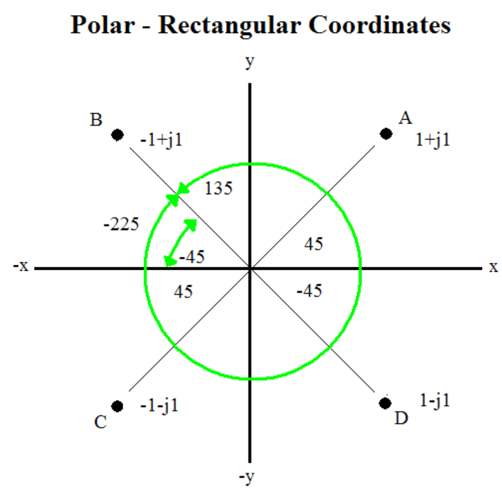


Figure 1

Table 1 Polar Coordinate Angle Convention					
Rectangular		Polar Coordinates			
R	jX	r	Formula Angle	Calculator Angle	Positive Convention
1	1	1.414	45	45	45
-1	1	1.414	-45	135	135
1	-1	1.414	-45	-45	315
-1	-1	1.414	45	-135	225

An easy way to resolve the angle dilemma is to add 180 degrees to the resulting polar angle if the real component of the rectangular coordinate is negative (e.g. -1+j1, -1-j1, etc). If the imaginary component is negative, then also add 360 degrees to the resulting polar angle.

Complex Number Multiplication

The key to complex number multiplication of rectangular coordinates is how the "j" operator is handled. Remember that "j" is equal to the square root of a negative one, so "j^2" is a negative one.

j = sqrt(-1).

And,

$j^2 = -1$.

Consider the following example. Multiple 3+j4 by 2+j3.

$$\begin{array}{rl} & 3 + j4 \\ * & \underline{2 + j3} \\ & 9j + 12j^2 \\ \underline{6 + 8j} & \\ 6 + 17j + 12j^2 & \end{array}$$

Since j^2 is equal to negative one, the equation can be simplified to,

6 + 17j + (12 * -1) or,

-6 + 17j

When multiplying polar coordinates, the resultant values are multiplied and the angle values are added. For example, what is the product of 5∠53.13 and 3.61∠56.31?

The resultant value is 5 * 3.61 or 18.05.

The angle is 53.13 + 56.31 or 109.44 degrees.

Therefore, the product of 5∠53.13 and 3.61∠56.31 is,

18.05∠109.44 which also can be expressed in rectangular coordinates as -6+j17.

The “α” Operator

The “α” operator is a short-hand method of representing a phase shift difference of 120 degrees. The “α” operator has a unity value at 120 degrees or:

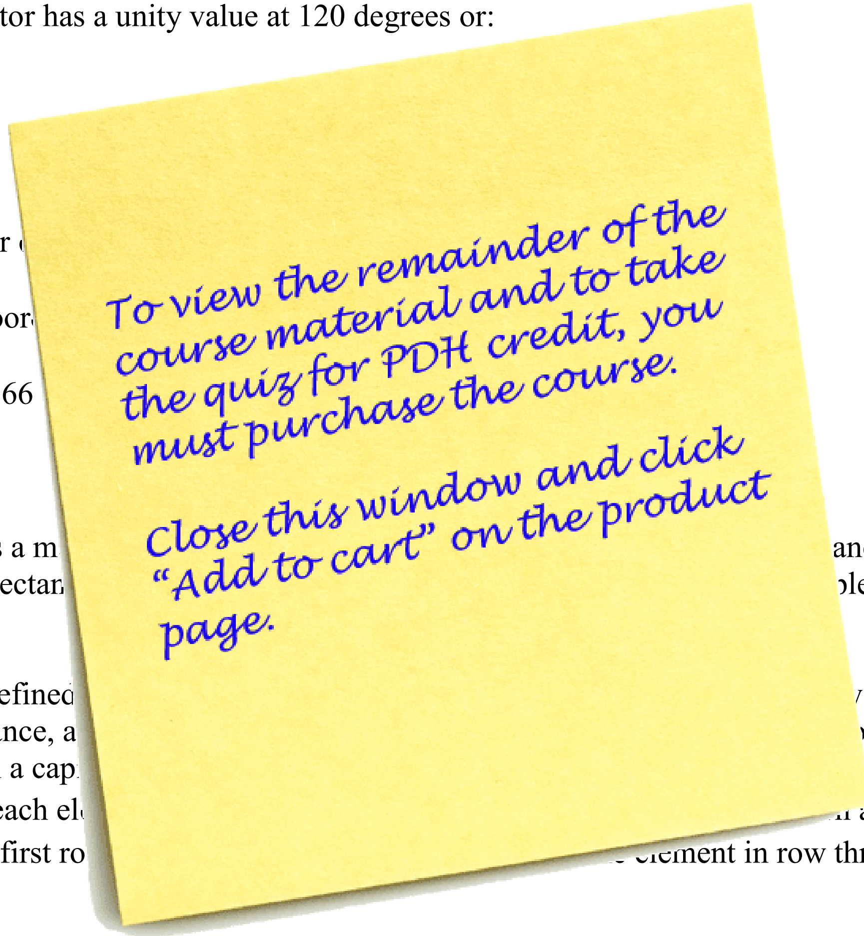
$\alpha = 1\angle 120.$

Similarly,

$\alpha^2 = 1\angle 240$ or

In rectangle coor

$\alpha = -0.5 + j0.866$



Matrices

Matrix math is a m aneous equations. A matrix is a rectan plex or a combination.

Matrices are defined s always listed first. For instance, a rmally identified with a cap trix are called elements and each el n as “a_{11}” for a number in the first ro element in row three and column two.

All of the following are valid matrices: