



Switching Transients Analysis Fundamentals

An Online Continuing Education Course for Engineers

Course Number: E-3049

Credit: 3 Hours / 3 PDH / 3 CPD

Switching Transients Analysis Fundamentals

Velimir Lackovic, Electrical Engineer

Power System Switching Transients

Introduction

An electrical transient occurs on a power system each time an abrupt circuit change occurs. This circuit change is usually the result of a normal switching operation, such as breaker opening or closing or simply turning a light switch on or off. Bus transfer switching operations along with abnormal conditions, such as inception and clearing of system faults, also cause transients.

The phenomena involved in power system transients can be classified into two major categories:

- Interaction between magnetic and electrostatic energy stored in the inductance and capacitance of the circuit, respectively
- Interaction between the mechanical energy stored in rotating machines and electrical energy stored in the inductance and capacitance of the circuit.

Most power system transients are oscillatory in nature and are characterized by their transient period of oscillation. Despite the fact that these transient periods are usually very short when compared with the power frequency of 50 Hz or 60 Hz, they are extremely important because at such times, the circuit components and electrical equipment are subjected to the greatest stresses resulting from abnormal transient voltages and currents. While over-voltages may result in flashovers or insulation breakdown, overcurrent may damage power equipment due to electromagnetic forces and excessive heat generation. Flashovers usually cause temporary power outages due to tripping of the protective devices, but insulation breakdown usually leads to permanent equipment damage.

For this reason, a clear understanding of the circuit during transient periods is essential in the formulation of steps required to minimize and prevent the damaging effects of switching transients.

Circuit Elements

All circuit elements, whether in utility systems, industrial plants, or commercial buildings, possess resistance, R , inductance, L , and capacitance, C . Ohm's law defines the voltage across a time-invariant linear resistor as the product of the current flowing through the resistor and its ohmic value. That is,

$$v(t) = Ri(t) \tag{1}$$

The other two elements, L and C , are characterized by their ability to store energy. The term "inductance" refers to the property of an element to store electromagnetic energy in the magnetic field. This energy storage is accomplished by establishing a magnetic flux within the ferromagnetic material. For a linear time-invariant inductor, the magnetic flux is defined as the product of the inductance and the terminal current. Thus,

$$\varphi(t) = Li(t) \quad (2)$$

where $\varphi(t)$ is the magnetic flux in webers (Wb), L is the inductance in henries (H), and $i(t)$ is the time-varying current in amperes (A). By Faraday's law, the voltage at the terminals of the inductor is the time derivative of the flux, namely,

$$v(t) = \frac{d\varphi}{dt} \quad (3)$$

Combining this relationship with Equation 2 gives the voltage-current relation of a time-invariant linear inductor as

$$v(t) = L \frac{di}{dt} \quad (4)$$

Finally, the term "capacitance" means the property of an element that stores electrostatic energy. In a typical capacitance element, energy storage takes place by accumulating charges between two surfaces that are separated by an insulating material. The stored charge in a linear capacitor is related to the terminal voltage by

$$q(t) = Cv(t) \quad (5)$$

where C is the capacitance in farads (F) when the units of q and v are in coulombs (C) and volts (V), respectively. Since the electrical current flowing through a particular point in a circuit is the time derivative of the electrical charge, Equation 5 can be differentiated with respect to time to yield a relationship between the terminal current and the terminal voltage. Thus,

$$i(t) = \frac{dq}{dt} \text{ or } i(t) = C \frac{dv}{dt} \quad (6)$$

Under steady-state conditions, the energy stored in the elements swings between the inductance and capacitance in the circuit at the power frequency. When there is a sudden change in the circuit, such as a switching event, a redistribution of energy takes place to accommodate the new condition. This redistribution of energy cannot occur instantaneously for the following reasons:

- The electromagnetic energy stored in an inductor is $E = \frac{LI^2}{2}$. For a constant inductance, a change in the magnetic energy requires a change in current. But the change in current in an inductor is opposed by an emf of magnitude $v(t) = L \frac{di}{dt}$. For the current to change instantaneously ($dt = 0$), an infinite voltage is required. Since this is unrealizable in practice, the change in energy in an inductor requires a finite time period.
- The electrostatic energy stored in a capacitor is given by $E = \frac{cV^2}{2}$ and the current-voltage relationship is given by $i(t) = c \frac{dv}{dt}$. For a capacitor, an instantaneous change in voltage $dt = 0$ requires an infinite current, which cannot be achieved in practice. Therefore, the change in voltage in a capacitor also requires finite time.

These two basic concepts, plus the recognition that the rate of energy produced must be equal to the sum of the rate of energy dissipated and the rate of energy stored at all times (principle of energy conservation) are basic to the understanding and analysis of transients in power systems.

Analytical Techniques

The classical method of treating transients consists of setting up and solving the differential equation or equations, which must satisfy the system conditions at every instant of time. The equations describing the response of such systems can be formulated as linear time-invariant differential equations with constant coefficients. The solution of these equations consists of two parts:

- The homogeneous solution, which describes the transient response of the system, and
- The particular solution, which describes the steady-state response of the system to the forcing function or stimulus.

Analytical solution of linear differential equations can also be obtained by the Laplace transform method. This technique does not require the evaluation of the constants of integration and is a powerful tool for complex circuits, where the traditional method can be quite difficult.

Transient Analysis Based on the Laplace Transform Method

Although they do not represent the types of problems regularly encountered in power systems, the transient analysis of the simple RL and RC circuits are useful illustrative examples of how the Laplace transform method can be used for solving circuit transient problems. Real-life circuits, however, are far more complicated and often retain many circuit elements in series-parallel combination even after simplification. These circuits will require several differential or integro-differential equations to describe transient behavior and must be solved simultaneously to evaluate the response. To do this efficiently, the Laplace transform method is often used.

LC Transients

General types of circuits that are described by higher-order differential equations are discussed. The double-energy transient, or LC circuit, is the first type of circuit to be considered. In double-energy electric circuits, energy storage takes place in the magnetic field of inductors and in the electric field of the capacitors. In real circuits, the interchange of these two forms of energy may, under certain conditions, produce electric oscillations. The theory of these oscillations is of great importance in electric power systems.

In the circuit shown in Figure 1 the circuit elements are represented with Laplace transform impedances. The response of the circuit to a step input of voltage due to the closing of the switch at $t = 0$ will be examined, assuming the capacitor is initially charged to the potential of $V_c(0^-)$ as shown.

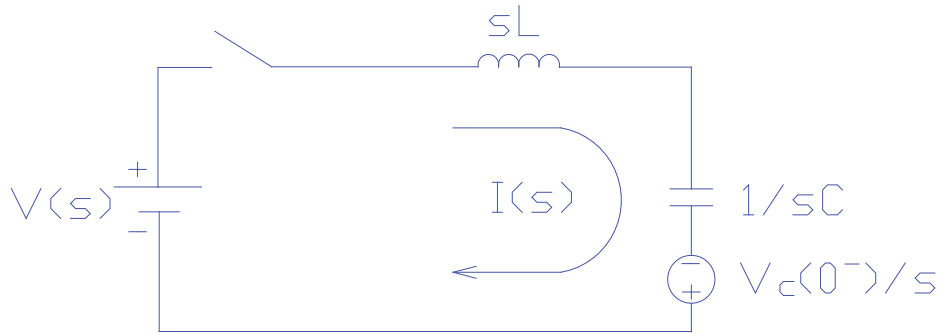


Figure 1. Double energy network

According to Kirchoff's voltage law, the sum of the voltages across all the circuit elements must equal the source voltage at all times. In equation form this is stated as

$$v(s) = I(s)s^2L - sLI(0) + \frac{I(s)}{c} - V_c(0) \quad (7)$$

Since there could be no current flowing in the circuit before the switch closes, the term $LI(0) = 0$. Solving for the current $I(s)$ in Equation 7,

$$I(s) = \frac{V(s)+V_c(0)}{L} \left[\frac{1}{(s^2+\omega_0^2)} \right] \quad (8)$$

where ω_0^2 is the natural frequency of the circuit, namely

$\frac{1}{\sqrt{LC}}$. From the table of inverse Laplace transforms, the transient response in the time domain is

$$i(t) = \frac{V(s)+V_c(0)}{Z_0} [\sin(\omega_0 t)] \quad (9)$$

where Z_0 is the surge impedance of the circuit defined by

$$Z_0 = \sqrt{\frac{L}{C}} \quad (10)$$

Clearly, the transient response indicates a sinusoidal current with a frequency governed by the circuit parameters L and C only.

Another interesting feature about the time response of the current in the circuit is that the magnitude of the current is inversely proportional to the surge impedance of the circuit Z_0 , which is a function of the circuit parameters of L and C .

In power system analysis, we are often interested in the voltage across the capacitor. Referring to Figure 1, the capacitor voltage is

$$V_c(s) = \frac{I(s)}{sC} - \frac{V_c(0)}{s} \quad (11)$$

where $V_c(0)$ is the initial voltage of the capacitor. Solving for $I(s)$ in the above equation results in

$$I(s) = sCV_c(s) + CV_c(0) \quad (12)$$

But because the current $I(s)$ is common to both elements L and C , we can substitute Equation 12 into Equation 8 to obtain the voltage across the capacitor. After rearranging the terms,

$$V_c(s) = \frac{V(s)\omega_0^2}{s(s^2+\omega_0^2)} + \frac{sV_c(0)}{s^2+\omega_0^2} \quad (13)$$

From the table of inverse Laplace transforms, the transient response is

$$v_c(t) = V(s)[1 - \cos(\omega_0 t)] + V_c(0) \cos(\omega_0 t) \quad (14)$$

The above equation is plotted in Figure 2 for various values of initial capacitor voltage $V_c(0)$.

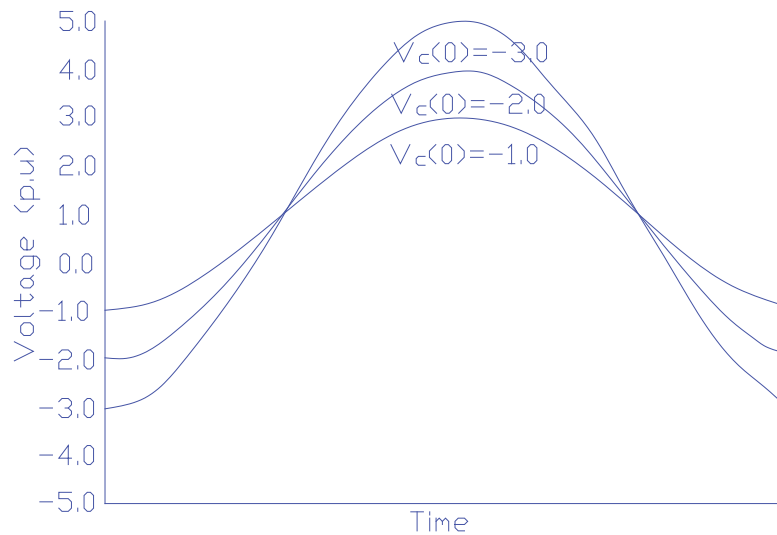


Figure 2—Capacitor voltage for various initial voltages

Examination of these curves indicates that, without damping, the capacitor voltage swings as far above the source voltage V as it starts below. In a real circuit, however, this will not be the case, since circuit resistance will introduce losses and will damp the oscillations. Treatment of the effects of resistance in the analysis of circuits is presented next.

Damping

Nearly every practical electrical component has resistive losses (I^2R losses). To simplify the calculations and to ensure more conservative results, resistive losses are usually neglected as a first attempt to a switching transient problem. Once the behavior of the circuit is understood, then

system losses can be considered if deemed necessary.

A parallel *RLC* circuit is depicted in Figure 3, in which the circuit elements are represented by their Laplace transform admittances. Many practical transient problems found in power systems can be reduced to this simple form and still yield acceptable results. With a constant current source, $I(s)$ and zero initial conditions, the equation describing the current in the parallel branches is

$$I(s) = sV(s)G + s^2CV(s) + \frac{V(s)}{L} \quad (15)$$

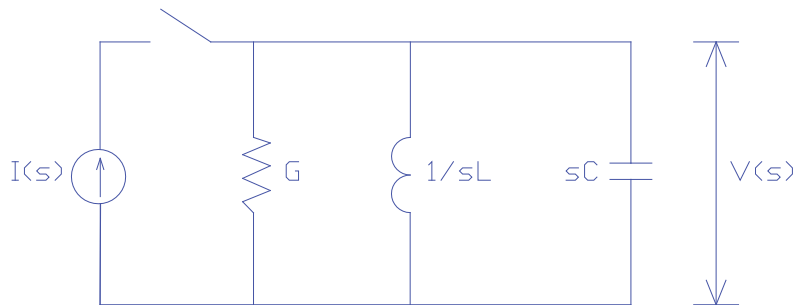


Figure 3—Parallel RLC circuit

Solving for the voltage results in

$$V(s) = \frac{I(s)}{C} \left(\frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \right) \quad (16)$$

Equation 16 can be written as

$$V(s) = \frac{I(s)}{C} \frac{1}{(s - r_1)(s - r_2)} \quad (17)$$

where r_1 and r_2 are the roots of the denominator:

$$r_1 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \quad (18)$$

At the natural resonant frequency, the voltage across the capacitor but opposite in phase to the voltage across the resistance in the circuit. The quality factor of the inductance or the capacitor of the circuit, is known as

$$Q_P = \frac{P_R}{P_T} \quad \text{or} \quad Q_P = \frac{1}{X} \quad (19)$$

To view the remainder of the course material and to take the quiz for PDH credit, you must purchase the course.
Close this window and click "Add to cart" on the product page.