Hydraulic Engineering

An Online Continuing Education Course for Engineers

Course Number: C-2038

Credit: 2 Hours / 2 PDH / 2 CPD

Hydraulic Engineering

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Introduction

Hydraulic engineering is a sub-discipline of civil engineering. It is concerned with the flow and conveyance of fluids both in pipes and open channels, principally water and sewage. For hydraulic engineers, it is important to understand: dimensions, units, and dimensional homogeneity, Understand benefits of dimensional analysis, , how to use the method of repeating variables (π Theorem), the concept of similarity and how to apply it to experimental modeling (Model Studies and Similitude), mass (continuity) Equation, Bernoulli's Equation, Open-channel flow, the Momentum Equation, the energy Equation, the gradually varied flow (GVF), the surface and bed slopes, the water surface profiles, and to Calculate the water surface curve length. Hydraulic engineering is mainly related to the design of channels, canals, and levees, and to both sanitary and environmental engineering. Also, it is the application of fluid mechanics principles to problems dealing with the control, transport, regulation, measurement, and use of water.

Dimensions and Units

For flow, each quantity has a dimension. The number of dimensions can be eliminated by using the so called "Fundamental dimensions". These are called as Mass, Length and Time or M, L and T respectively. In the SI system (Systemé International), they are measured in Kg, meter and s. while for the Imperial system, Pound, Feet and s. Table 1 shows some of the quantities that are being used in this course.

Dimensional Analysis

Dimensional analysis presents a method for reducing complex physical problems to the simplest (that is, most efficient) form prior to obtaining a quantitative answer for certain phenomena. For dimensional homogeneity, each additive term must have the same dimensions (e.g. Bernoulli's equation):

$$\frac{P}{\rho} + \frac{V_s^2}{2} + gz = const$$

Any equation must be homogeneous dimensionally. We cannot give the final relation between the groups of variables without the knowledge of their dimensions. For dimension analysis, there are two main methods: Rayleigh's method and Buckingham method. Don't use Rayleigh's method unless it has been mentioned clearly in the question. In Rayleigh's method, to get a solution without symbols, the variables must not exceed 4 including the main variable.

<u>Example:</u>

v=f(L, g, ρ)

$V=k(L)^a (g)^b (\rho)^c$

$\frac{L}{T} = k\ (L)^a\ (\frac{L}{T^2})^b\ (\frac{M}{L^3})^c$

For

L: 1=a+b-3c

T: 0=c

M:-1=-2 b

A=1/2, b=1/2, c=0 → v=k$\sqrt{Lg}$

Buckingham π Theorem

The Buckingham π theorem is a key theorem in dimensional analysis. It is a formalization of Rayleigh's method of dimensional analysis. To obtain the dimensionless groups π_i, select some repeating variables r that contain some primary dimensions (L, M, T) but do not form dimensionless groups. The π_i parameters are formed by combining each of the remaining m - r parameters with the repeating variables: $\pi_i = m - r$

Table 1: Physical Quantities and Their Units

Quantity	symbol	SI measurement units	unit dimensions
Length	L	m	L
Area	A	m2	L2
Volume	V	m3	L3
Velocity	u	$m.sec^{-1}$	LT^{-1}
Speed of sound	a	$m.sec^{-1}$	LT^{-1}
Volume flow rate	Q	M^3/sec	L^3T^{-1}
Mass flow rate	m	Kg/sec	MT-1
Pressure, stress	P, σ	N/m^2 (Pa)	$ML^{-1}T^{-2}$
Strain rate	e	Sec^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω	Sec^{-1}	T^{-1}
Dynamic viscosity	μ	$N. sec/m^2$	$ML^{-1}T^{-1}$
Kinematic viscosity	ν	m^2/sec	L^2T^{-1}
Surface tension	σ	N/m	MT^{-2}
Force	F	N	$M LT^{-2}$
Moment, torque	M	N m	$M L^2T^{-2}$
Power	P	N m/sec (Watt)	ML^2T^{-3}
Work, energy	W,E	N m (Joule)	ML^2T^{-2}
Density	ρ	Kg/m^3	$M L^{-3}$
Temperature	T	T	θ
Specific heat	C_p, c_v	$m^3/ T^2 sec^2$	$θ^{-1} L^2T^{-2}$

Thermal conductivity	κ	Kg m sec^3/T	θ^{-1} MLT^3
Coefficient of thermal expansion	β	T^{-1}	θ^{-1}

For most problems r = n (# of primary dimensions). In some cases, r < n, which occurs when repeating parameters (r) are dependent on other variables (i.e., they are a combination of other repeating parameters).

Example 1: Let's say we want to know the relationship of the effect on pressure drop (ΔP) of the variables d, L, ρ, μ and v.

Answer:

f (ΔP, d, L, ρ, μ, v) = O

No. of variables = n = 6 (That are: ΔP, d, L, ρ, μ, v)

No. of fundamental dimensions = m = 3 (That are: [M], [L], [T])

By Buckingham's theorem,

No. of dimensionless groups = n -m = 6-3 = 3

The recurring set must contain three variables that cannot themselves be formed into a dimensionless group. In this case, there are two restrictions:

- Both L and d cannot be chosen as they can be formed into a dimensionless group, (L /d).
- ΔP, ρ, and v cannot be used since ($\Delta P / \rho v^2$) is dimensionless.

The variables d, v and ρ are chosen as the recurring set.

The dimensions of these variables are:

d = [L]

v = [LT^{-1}]

ρ = [ML^{-3}]

Rewriting the dimensions in terms of the variables chosen:

[L] = d

$[M] = \rho\, d^3$

$[T] = dv^{-1}$

The dimensionless groups are formed by taking each of the remaining variables ΔP, L and μ in turn:

ΔP has dimensions of $[ML^{-1}T^{-2}]$

Therefore $\Delta P\, M^{-1}\, L\, T^2$ is dimensionless

Substituting the dimensions in terms of variables

$\pi_1 = \Delta P\, (\rho\, d^3)^{-1}\, (d)\, (dv^{-1})^2$

$= \Delta P/ \rho\, v^2$

L has dimensions of [L]

$L\, [L]^{-1}$ is therefore dimensionless

$\pi_2 = L/d$

μ has dimensions of $[ML^{-1}T^{-1}]$

$\mu\, [M^{-1}LT]$ is therefore dimensionless

$\pi_3 = \mu\, (\rho\, d^3)^{-1}\, (d)\, (dv^{-1})$

$= \mu/dv\rho$

Therefore,

$f\, (\Delta P/ \rho\, v^2,\ L/d,\ \mu / v\, \rho\, d)$

$\Delta P / \rho\, v^2 = f\, (L/d,\ v\, \rho\, d/\mu)$

Example 2: Let's say we want to study the unsteady force induced by the flow on the cylinder (see Figure 1) which is a function of F, V_∞, μ, ρ, D, t :

- Notice that there are 6 independent variables, all in dimensional form
- By means of dimensional analysis (Buckingham π Theorem), the dependence between the variables is reduced to 3:

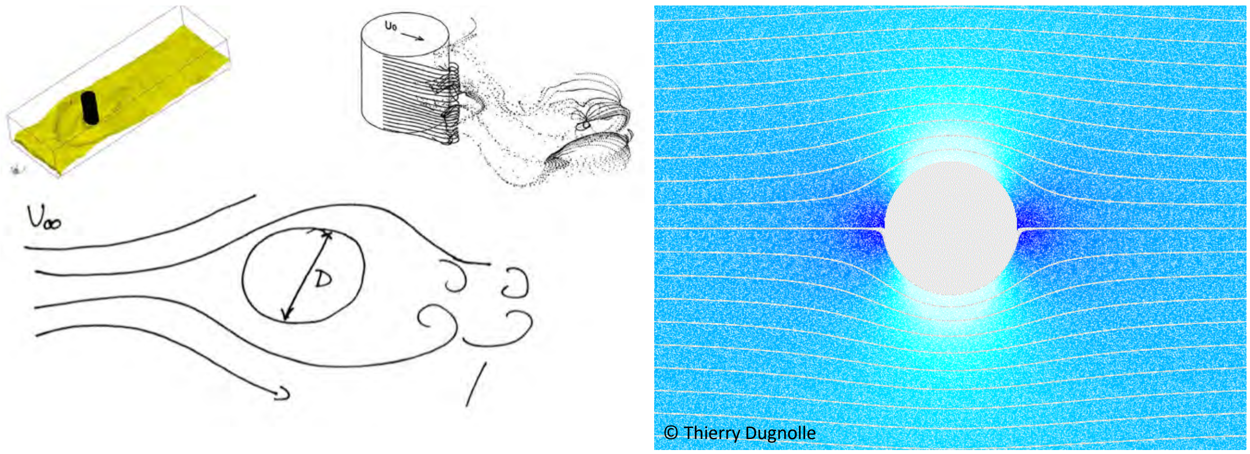

Figure 1: Flow past a cylinder

- Number of independent variables or physical parameters:

$F, V_\infty, \mu, \rho, D, t \rightarrow m = 6$

- Number of primary dimensions:

$M, L, T \rightarrow n = 3$

- number of repeating variables

$r = n = 3$

- V_∞, ρ, D w
- Number o

$\pi_i = m - r = 6 -$

- Finding the