Engineering Economics - Fundamentals

An Online Continuing Education Course for Engineers

Course Number: BS-2020

Credit: 2 Hours / 2 PDH / 2 CPD

Engineering Economics - Fundamentals

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Overview

The "dollars and cents" business side of Engineering often doesn't receive the emphasis that employers would like. While engineers typically work towards a goal of finding the most efficient solution to a problem, in the business world, "efficiency" often means the least-cost solution. Engineering Economics provides the tools for determining the most cost-effective alternative in an organized fashion.

In this course, the student will learn some of the fundamentals of Engineering Economics. This topic is relevant to any engineer who must consider cost in his or her decisions.

After completing this course, the student should understand time value of money calculations, the effects of inflation, the accounting concept of depreciation, and how to recognize and organize project cash flows.

Specific Knowledge or Skill Attained

This course teaches the following specific knowledge and skills:

1) Time Value of Money

2) Discount Rate

3) Inflation

4) Depreciation

5) Project Cash Flows.

Course Content

1.0. Time Value of Money

Understanding the time value of money is the central theme of Engineering Economics. **Interest** is the cost of borrowing money from a lender (e.g., a bank or other financial institution, or an individual) Even if an owner doesn't take on debt to finance a project, the opportunity cost of not investing the funds elsewhere is considered in the economic analysis of the project. This interest rate equivalent is often called the **discount rate**.

1.1. Future Value with Compound Annual Interest Calculation

Virtually every commercial lending transaction makes use of compound interest. The general form of the Future Value equation takes this into consideration

$$\mathbf{FV = P\,(1+i/t)^{nt}}$$

Where FV = Future Value
P = initial Principal
i = effective interest rate
n = term of the investment in years
t = number of times interest is compounded during year

Example 1: What is the Future Value of \$1,000 invested at 8% interest compounded annually for 20 years?

$$\mathbf{FV = P\,(1+i/t)^{nt}} = \$1{,}000\,(1 + (0.08/1))^{(20 \times 1)} = \$4{,}660.96$$

Interest may be compounded monthly, daily, hourly, or even continuously

Compounded monthly: $FV = \$1{,}000\,(1 + (0.08/12))^{(20 \times 12)} = \$4{,}926.80$
Compounded daily: $FV = \$1{,}000\,(1 + (0.08/365))^{(20 \times 365)} = \$4{,}952.16$
Compounded hourly: $FV = \$1{,}000\,(1 + (0.08/8760))^{(20 \times 8760)} = \$4{,}953.00$

As the period for compounding becomes infinitely small (i.e. continuous compounding), the Future Value formula is expressed as:

$$\mathbf{FV = Pe^{in}}$$

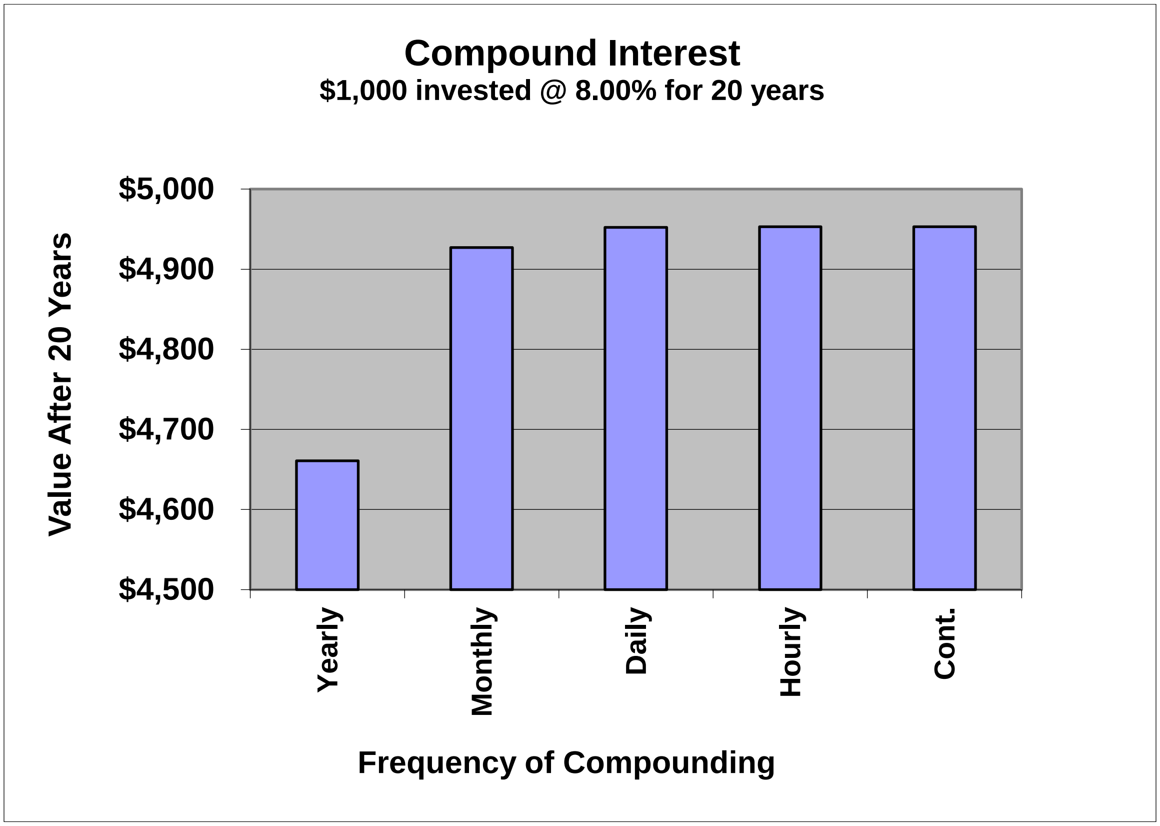
Where FV = Future Value
P = initial Principal
e = constant (2.71828183)
i = effective interest rate
n = term of the investment in years

(Note: **e** is a constant and the base of the natural logarithm. Mathematicians often call it Euler's (sounds like "oiler") Number in honor of the Swiss mathematician Leonhard Euler. Financial types usually just call it the "compound interest constant".)

Example 2: What is the Future Value of \$1,000 invested at 8% interest compounded continuously for 20 years?

$$\mathbf{FV = Pe^{in}} = \$1{,}000\,(2.71828)^{(0.08 \times 20)} = \$4{,}953.03$$

The effect of increasing the frequency of compounding is depicted graphically in the following chart. Over a 20-year term, there's little increase in the end value of the investment if interest is compounded more frequently than daily.



1.2. Nominal and Effective Rates of Interest

The phenomenon of compound interest may also be considered by calculating the **effective** rate of interest. In Examples 1 and 2 above, the **nominal** rate is 8%. The effective rate of interest grows as the period of compounding shortens. The formula for the effective rate of interest is as follows.

$$\mathbf{i = (1+ ((i^{(t)})/t))^{t}-1}$$

Where i = effective interest rate
$i^{(t)}$= nominal interest rate
t = number of times interest is compounded during year

Note that in the term $\mathbf{i^{(t)}}$**,** (t) is a superscript, not "i" to the "t" power. The variable for the nominal rate of interest with monthly compounding would be denoted $\mathbf{i^{(12)}}$. For daily compounding, the variable would be $\mathbf{i^{(365)}}$

Example 3: What is the effective rate of interest for an 8% nominal annual rate of interest compounded annually?

$$\mathbf{i = (1+ ((i^{(t)})/t))^t - 1} = (1+ ((0.08)/1))^1 - 1 = 0.0800000 = \mathbf{8.00\%}$$

In the case of annual compounding, the nominal and effective rates of interest are the same.

Example 4: What is the effective rate of interest for an 8% nominal annual rate of interest compounded monthly?

$$\mathbf{i = (1+ ((i^{(t)})/t))^t - 1} = (1+ ((0.08)/12))^{12} - 1 = 0.0829995 = \mathbf{8.30\%}$$

As we increase the frequency of compounding from daily to hourly to an infinite number of periods (i.e. continuous compounding), the effective interest rate equation becomes

$$\mathbf{i = e^{i^{(t)}} - 1}$$

Where i = effective interest rate
$i^{(t)}$= nominal interest rate (where t approaches infinity)
e = constant (2.71828183)
t = number of times interest is compounded during year

Example 5: What is the effective rate of interest for an 8% nominal annual rate of interest compounded continuously?

$$\mathbf{i = e^{i^{(t)}} - 1}$$

$$\mathbf{i} = (2.71828183)^{0.08} - 1 = 0.083287 = \mathbf{8.33\%}$$

1.3. Average Annual Return vs. Average Annualized Return

Many people don’t realize that there’s a big difference between average annual returns and average *annualized* returns. Average annual return usually provides a poor description of investment performance.

Average annual return is simply the sum of the annual change in the value of an investment divided by the number of years.

$$\mathbf{R_{ave} = \left(\sum R_n\right) / n}$$

Where R_{ave} = Average Annual Return
R_n = Annual Return for Year (n)
n = term of the investment in years

Let's look at two examples.

Example 6: What is the average annual return for a $1,000 investment that gains 30% the first year, loses 30% the second year, and then gains 30% the third year? How much money do you have at the end of three years?

$\mathbf{R_{ave} = (\sum R_n) / n}$ = (30%+(-30%)+30%)/3 = **10%**

	Starting Balance	Annual Return (%)	Annual Return ($)	Year End Balance
Year 1	$ 1,000	30%	$ 300	$ 1,300
Year 2	$ 1,300	-30%	$ (390)	$ 910
Year 3	$ 910	30%	$ 273	**$ 1,183**

Example 7: What is the average annual return for a $1,000 investment that gains 10% per year for three years? How much money do you have at the end of three years?

$\mathbf{R_{ave} = (\sum R_n) / n}$ = (10%+10%+10%)/3 = **10%**

	Starting Balance	Annual Return (%)	Annual Return ($)	Year End Balance
Year 1	$ 1,000	10%	$ 100	$ 1,100
Year 2	$ 1,100	10%	$ 110	$ 1,210
Year 3	$ 1,210	10%	$ 121	**$ 1,331**

1.4. Compound Annual Growth Rate (CAGR)

Because of the inconsistencies illustrated in the two examples above (i.e., an investment with the same average annual return had very different values after three years), Annualized Annual Return (often referred to as Compound Annual Growth Rate (CAGR)) is the preferred measure of investment performance.

$$\mathbf{CAGR = (FV / P)^{(1/n)} - 1}$$

Where CAGR = Compound Annual Growth Rate
FV = Future Value
P = initial Principal
n = term of the investment in years

Example 8: What is the CAGR for the $1,000 investment described in Examples 6 and 7 above?

For Example 6: $\mathbf{CAGR = (FV / P)^{(1/n)} - 1}$ = $(\$1,183 / \$1,000)^{(1/3)} - 1$ = **5.76%**

For Example 7: **CAGR = (FV / P)$^{(1/n)}$ – 1 =** (\$1,331 /\$1,000)$^{(1/3)}$ – 1 = **10.0%**

1.5. Present Value of a Series of Uniform Payments

Present Value is usually expressed as a factor for a series of uniform payments of \$1.

$$\mathbf{PV = ((1+i)^n – 1) / (i(1+i)^n)}$$

Where PV = Present Value
i = interest rate
n = term of the investment in years

Example 9: What is the Present Value for a uniform series of 10 annual payments of \$1,000 at 7% interest?

PV = ((1+i)^n – 1) / (i(1+i)^n) = ((1+0.07)^{10} – 1) / (0.07(1+0.07)^{10}) **= 7.024**

PV= \$1,000 x 7.024 = \$7,024

You can also use the Present Value function in Microsoft Excel to make this calculation

PV(rate,nper,pmt) = PV(0.07,10,1000) = 7,024

1.6. Amortizing a loan

The word "amortize" is derived f ... h. Amortizing a loan literally mean "killi

We can use the Prese ... required to retire a debt.

Example 10: What is ... 000 loan at 8% interest, compounded annually

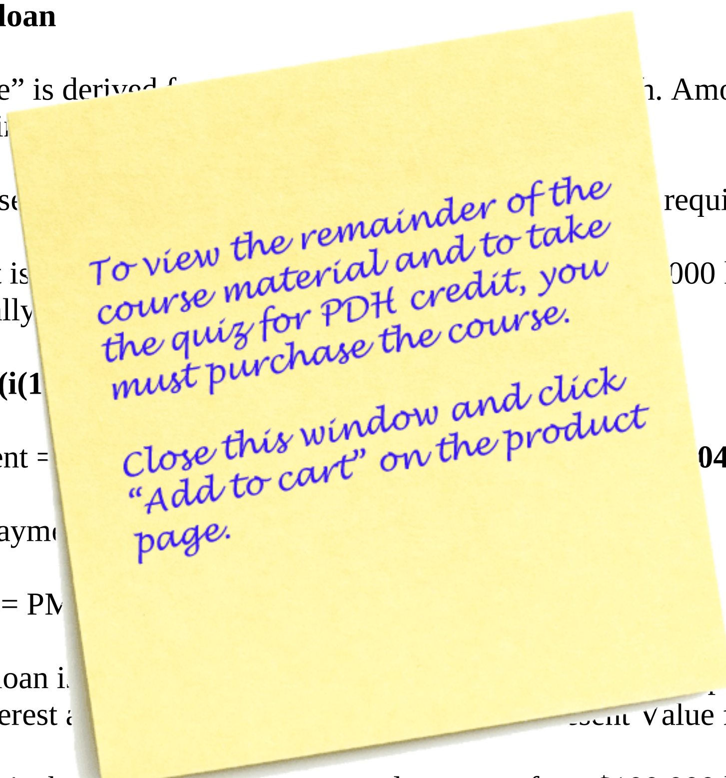
**PV = ((1+i)^n – 1) / (i(1

Annual loan payment = ... 046**

You can also use Paym

PMT(rate,nper,pv) = PM

If the interest on a loan i ... y calculate the effective rate of interest a ... Value formula.



Example 11: What is the ... uniform annual payment for a \$100,000 loan at 8% interest, compounded daily, to be repaid over 5 years?